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A COMMENTARY ON CHAPTER TEN OF *SCIENCE AND THE MODERN WORLD* BY ALFRED NORTH WHITEHEAD

Chapter 10 of *Science and the Modern World*[1] contains Whitehead's most complete analysis of the "ordering of the eternal objects" to which he refers frequently in *Process and Reality*.[2] It also contains a profound analysis of the relationship between possibility and actuality. On the other hand, it strongly reflects Whitehead's mathematical background, and it is notoriously difficult to read and understand.

I have found this chapter to be profoundly illuminating, and I have written this commentary[3] in an attempt to make it more accessible.

ABSTRACTION

> *These metaphysical chapters are purely descriptive. Their justification is to be sought, (i) in our direct knowledge of the actual occasions which compose our immediate experience, and (ii) in their success as forming a basis for harmonizing our systematized accounts of various types of experience, and (iii) in their success as providing the concepts in terms of which an epistemology can be framed. By (iii) I mean that an account of the general character of what we*

> *know must enable us to frame an account of how knowledge is possible as an adjunct within things known.* (158)

We start this chapter with the idea of actual occasions firmly in mind. Reality is to be understood as a vast society of actual occasions, or causally interacting drops of experience. The purpose of this chapter (the two chapters mentioned also include the subsequent chapter, called "God") is an attempt to elucidate the metaphysical presuppositions in terms of which a universe of actual occasions makes sense. We begin this essay with certain presuppositions:

- It is assumed that we have direct knowledge (both from within and from without) of these actual occasions
- It is assumed that an ontology based on actual occasions allows us to form a harmonious understanding of ourselves and of the world.
- It is assumed that an account of experience in terms of actual occasions enables us form an adequate account of *how* it is that we can know what we know. The failure of materialism to answer this crucial epistemological question is a great scandal of modern thought.

> *In any occasion of cognition, that which is known is an actual occasion of experience, as diversified by reference to a realm of entities which transcend that immediate occasion in that they have analogous or different connections with other occasions of experience.* (158)

The notion of "diversification," which is mentioned in this paragraph, is first introduced in *An Enquiry Concerning the Principles of Natural Knowledge*, where Whitehead says: "our perceptual knowledge of nature consists in the breaking up of . . . the ultimate experienced fact. This whole is discriminated as being a complex of related entities, each entity having determinate qualities and relations and being a subject concerning which our perceptions, either directly or indirectly afford definite information. This process of breaking up the subject matter of experience into a complex of entities will be called the 'diversification of nature'"[4] "The ultimate experienced fact" is the experience of an actual occasion. It is the universe as apprehended by a subject situated in space and time. It is what we sometimes call the "specious present." The specious present includes all of the universe as it is experienced from a specific place and over an interval of time. It is what William James called a "drop of experience," and what Whitehead, in his earlier work, described as a "duration."

As this experience presents itself to us in perception, it is "diversified," which is to say that:

- We, and all other actual occasions, can discriminate within each moment of our experience a multiplicity of constituent events.
- Each of these events has certain qualities, and is in certain relations with other entities. The qualities in terms of which we know events are *abstractions* from the events, and are here designated *eternal objects*. Much of this essay concerns the nature of eternal objects and of the relations that they form to each other and to actual occasions.

- Our perceptions, directly or indirectly, inform us as to the nature of these discriminated events.

For example, if we analyze a child's ball, we might begin by noting the ball as one of the events taking place in our specious present. *It* is *there*. Then, by means of our perceptual process, we might notice various eternal objects implicated in it – eternal objects such as "ball," "red" and "round." Thus our perceptual experience is "diversified" into events and objects.

I would like to bring attention to the significance of this point. Particularly since the time of Descartes, and the ontological distinction that he drew between extended substance and thinking substance, it has been difficult to understand how our thoughts in general, and our mathematical reasoning in particular, actually correspond to the realities of the world. Some of the preconditions for Whitehead's answer to this difficulty are established here.

- Our experience of the world can be diversified, which is to say that our experiences can be analyzed into events and the abstract factors (eternal objects) characterizing them.
- We can form an adequate description of the whole of our experience in terms of the events and eternal objects thus diversified.
- The eternal objects together constitute a 'realm of possibility,' (as will be presently discussed) and all actualities arise, in one sense, out of this realm.

It will emerge in what follows that an adequate epistemology can be constructed on this basis.

Returning to the paragraph at hand, Whitehead points out that an important character of the eternal objects which emerge in the operation of diversification is that they transcend the particular events from which they are abstracted.

> *For example a definite shade of red may, in the immediate occasion, be implicated with the shape of sphericity in some definite way. But that shade of red, and that spherical shape, exhibit themselves as transcending that occasion, in that either of them has other relationships to other occasions.* (158)

In our example, the ball is an event that is red and round, and these two entities ("redness" and "roundness,") are together in the ball. However, every event refers, in its nature, to other events with which it is causally implicated. Also, there are events that are not balls, balls that are round but not red, and events (like fire engines) that are red but not round. So "redness" and "roundness" transcend the particular occasion of experience from which they are abstracted in that it is always possible for them to be implicated in other events as well.

> *Also, apart from the actual occurrence of the same things in other occasions, every actual occasion is set within a realm of alternative interconnected entities. This realm is disclosed by all the untrue propositions which can be predicated significantly of that occasion. It is the realm of alternative suggestions, whose foothold in actuality transcends each actual occasion. The real relevance of untrue propositions for each actual occasion is disclosed by art, romance, and by criticism in reference to ideals.* (158)

Consider the red ball which we have been discussing. Any actual ball will be somewhere in space and somewhen in time. It will arise in some specific universe which constitutes its actual context. But the actual world out of which it arises is only one dimension of its context. It also emerges out of a domain of possibility. It might, given a different past, have been blue. It might have been larger or smaller. It might, even having the same past, have given way to the pressure within it, and have torn and collapsed. All of these "might have beens" must be taken into account if we are to appreciate the full character that the ball does exhibit in this place and at this moment. This ball is also a locus for possibilities stretching off into the future. It might tear and collapse in the next moment, or it might be the occasion for a friendly game of "catch." The point is that every event which we discern in our experience is situated within a domain of possibilities – possibilities that might have been chosen, but were not, and possibilities which may be chosen in the present. It is an interesting exercise to contemplate some ordinary object in our environment, and to "feel" that it is not only situated in spacetime, but is also situated in a "space" of possibility.

Here Whitehead points to that space of possibility by referring to all of the untrue propositions which can be significantly predicated of an occasion. The point is that these untrue propositions are not meaningless, rather they point to a domain of alternative possibilities which has genuine ontological significance. This significance is revealed in "art, romance, and by criticism in reference to ideals," all of which explore how things might have been, how things might be, and how things should be. Without this realm of

"might have been," "might be," and "should be," the actual character of "what is" cannot be fully appreciated.

> *It is the foundation of the metaphysical position which I am maintaining that the understanding of actuality requires a reference to ideality. The two realms are intrinsically inherent in the total metaphysical situation. The truth that some proposition respecting an actual occasion is untrue may express the vital truth as to the aesthetic achievement. It expresses the 'great refusal' which is its primary characteristic. An event is decisive in proportion to the importance (for it) of its untrue propositions: their relevance to the event cannot be disassociated from what the event is in itself by way of achievement.* (158)

The Western philosophical tradition has involved, from the time of Aristotle, a discussion of the relations between Forms and Substances. In modern times, there has been a tendency to assume that the only forms that are truly effective are mathematical forms functioning as natural laws. Qualitative forms have tended to retreat (as "secondary qualities") into the shadowy recesses of the subject. One of Whitehead's projects is the rehabilitation of all Forms, or "eternal objects" – both qualitative and quantitative. Whitehead envisions the eternal objects as functioning in the creative advance in two different ways. First, they exist as potentials, possibilities, lures for feeling, or ideals. In this mode, each eternal object is a potential form of definiteness that might, given the proper motivation and the proper circumstances, become an element in the character of a new occasion of experience. In their second mode of functioning, eternal objects do in fact characterize occasions that have become fully actual and are

now in the settled past. Thus eternal objects are both shapes of possibility and shapes of settled fact. Part of the purpose of this particular chapter is to show how thoughts concerning both quality *and* quantity are amenable to systematic thought and also relevant to the structure of the world.

It is important, in this context, to understand how Whitehead envisions the relationship between potentiality and actuality. The situation, or world, out of which actual occasions arise never fully determines the character that those occasions will have. This is the truth of indeterminacy which quantum mechanics has so forcibly brought to the attention of modern thought. Every event, or actual occasion, grows out of a field of possibilities many of which are contraries, so that it is impossible to realize all of them at the same time and in the same place. In its coming to be, each event must choose among incompatible possibilities so that it grows towards a coherent, complete and fully definite "aesthetic synthesis of possibilities." This growth towards definiteness is governed by both logical and aesthetic criteria, and it strives towards the maximization of value in itself and in its relevant future. In the synthesis that comes to characterize an occasion of experience, many elements must be rejected as not fitting. This rejection is its 'great refusal.' Of course, there are always some possibilities that it does affirm. When it has made a perfectly determinate relationship to all of the eternal objects, its character is fully definite. With the achievement of that definiteness, it becomes actual. Actuality emerges out of possibility by a process of decision among posibilities, and thus, "actuality requires a reference to ideality."

Some occasions of experience barely entertain those possibilities which they reject. Some occasions of experience actively entertain alternative possibilities, and deliberately choose among them. These occasions are, as Whitehead says, " decisive." Their importance, for themselves and for the world, is measured as much by the alternative that they reject as by those that they accept.

> *These transcendent entities have been termed 'universals.' I prefer to use the term 'eternal objects,' in order to disengage myself from presuppositions which cling to the former term owing to its prolonged philosophical history.* (159)

The history of the term "universals" is a significant part of the history of Western philosophy, and it is too long and complex to enter into here. A few words about some of the distinct features of Whitehead's doctrine may, however, be useful.

First, Whitehead's doctrine resembles that of Aristotle in that his eternal objects are actually 'ingredient' in experience and in the objects of experience. When I experience a ball as round, roundness is actually ingredient in the ball. Also, like Aristotle, Whitehead sees that we can discern eternal objects by abstraction from our experience. On the other hand, Whitehead's doctrine resembles that of Plato in that he asserts the existence of eternal objects that have never characterized any events in the universe. This is to permit evolutionary emergence. For example, according to the current doctrines of scientific cosmology, the form of Helium was not present in our universe until Hydrogen atoms formed vast clouds that ignited as Suns. Also, the forms through which living and thinking beings apprehend the world were not present in this universe

until the evolutionary emergence of life and cognitive mind. The possibilities that may characterize future actual occasions are, to a great extent, unknown, but whatever will be possible must, in some sense, always and already be possible. This does not mean, however, that the eternal objects exist, as Plato held, in a pure realm of their own. Rather they are, in their very natures nothing but possibilities that may, given the right circumstances, characterize actualities.[5]

Aristotle, and many other philosophers, often use "universals" as the complement to "particulars," so that universal may characterize particulars, but particulars never characterize each other. Whitehead also rejects that position. In his understanding, all past particulars (actual occasions) are ingredient in, and so lend their characters to, every new actual occasion that arises.

Also, as Whitehead observes later in this essay, the term "universal" has, historically, been associated with an ontology rooted in the activity of classification. Classification was one of the greatest discoveries of the Greek genius. The Greeks realized that it is possible to identify certain dominant characteristics of the natural entities of the world, and that those characters can be ordered into a logical hierarchy. For example, I might divide the universe of natural entities into minerals (which do not grow and do not move on their own), plants (which grow but do not move on their own), and animals (which both grow and move on their own). Then, within the domain of animals, I might roughly distinguish the two-legged (birds and humans), the four-legged (most animals), and the eight-legged (spiders). Now, whenever I find an unfamiliar animal, I can classify it by its number of legs. For example, let us say that I have established empirically, that

all spiders spin webs. Now, I encounter an insect that I have never seen before. Upon counting its legs and finding exactly eight of them, I conclude that this is a spider. Immediately, without ever seeing this particular insect spin a web, I know that it will do so. The classification of natural entities is an immense gain in knowledge of the world. This leads, however, to the notion that natural entities can be fully characterized by the "universals" on which classification is based. Whitehead, as we will see, entirely rejects this as an adequate way to express the full character of actual entities.

For all of these reasons, Whitehead avoids the term "universals" and uses, instead, the term "eternal objects."

> *Eternal objects are thus, in their nature, abstract. By 'abstract' I mean that what an eternal object is in itself – that is to say, its essence – is comprehensible without reference to some one particular occasion of experience. To be abstract is to transcend particular concrete occasions of actual happening. But to transcend an actual occasion does not mean being disconnected from it. On the contrary, I hold that each eternal object has its own proper connection with each such occasion, which I term its mode of ingression into that occasion. Thus an eternal object is to be comprehended by acquaintance with (i) its particular individuality, (ii) its general relationships to other eternal objects as apt for realization in actual occasions, and (iii) the general principle which expresses its ingression in particular actual occasions.* (159)

With this passage, we begin the process of exploring the nature of eternal objects. It is the prerogative of mind to comprehend eternal objects in themselves without making reference to some particular occasion of experience in which they are ingredient. Also, (as we will see below) each eternal object is implicated in relations with other eternal objects and mind has the capacity to trace these relations (to "think"), also without reference to specific actualities. By thinking, mind can discern eternal objects that have never been actualized in experience. However, even though mind can comprehend eternal objects and relations among eternal objects without reference to specific actualities, this does not mean that eternal objects are not intrinsically connected to the world of actuality. In fact each and every eternal object has a definite relationship to each and every occasion of experience in the creative advance. The connection that each eternal object makes with any given occasion of experience is its "mode of ingression" in that occasion. The notion of "mode of ingression" requires some explanation.

Whitehead uses two words to describe the various ways in which eternal objects have ingression into actual occasions. The first of these is its "mode." Actual occasions, in the process of their coming-to-be (their "concrescence") are subjective beings who remember, anticipate, and think.[6] Each of these – "memory", "anticipation," and "thought" is a "mode" in which eternal objects ingress into occasions. As a result of its subjective process, each occasion comes to have a definite character through which it will be known to future occasions. This is another mode of ingression, the mode of full ingression as character. It is important to note that "memory," which appears in this essay as analogous to anticipation and thought, becomes, in *Process and Reality*, assimilated to efficient cause. That is, the process through which I perceive the objects

around me in the world (which are, as I perceive them, already in the past), and the process through which I perceive my own past experiences, are recognized as the same process. Whitehead had not yet formed that doctrine at the time of the writing of this essay.

The second way in which ingression is described is in terms of "grade." The grade of ingression of a particular eternal object in a given actual occasion is just the importance of that eternal object in the final aesthetic synthesis of the occasion. The occasion may definitely exclude a particular eternal object from its aesthetic synthesis, it may definitely include it in its aesthetic significance, it may include it as an unrealized but considered possibility, as a minor characteristic and so forth.

> *These three headings* [i.e., (i), (ii) and (iii) in the previous paragraph from Whitehead's text – ew] *express two principles. The first principle is that each eternal object is an individual which, in its own peculiar fashion, is what it is. This particular individuality is the individual essence of the object, and cannot be described otherwise than as being itself. Thus the individual essence is merely the essence considered in respect to its uniqueness.* (159)

This part of the first principle refers to the first of the three headings above, namely to the doctrine that "each eternal object can be comprehended by reference to its particular individuality." Recall that the world of experience is open to thought by virtue of the fact that it can be decomposed, factored, or diversified into plurality of abstractions in terms of which it can then be understood. Also, these factors transcend the immediate

experiences from which they are abstracted, and mind can think by tracing relations among them. But what are eternal objects considered in themselves? The first thing we can notice about these eternal objects is that each one is just exactly what it is. This is particularly clear in the case of simple eternal objects, such as a particular shade of red, or a particularly auditory note. With these simple eternal objects, analysis reaches an end. We can recognize a particular shade of red, we can compare various events in terms of the presence or absence of this shade, but this shade of red is just what it is, and there is nothing else to be said about it.[7] This is the case even with more complex eternal objects which can be analyzed into simpler elements. Take the eternal object characterizing the red ball we have been examining. The definition of the ball can make reference to roundness, redness and other simple eternal objects, but the particular relationship among them by virtue of which they are a ball is, itself, a unique eternal object that is, in the end, just what it is. This sheer givenness of the individual characters of eternal objects is a precondition for thought.

> *Further, the essence of an eternal object is merely the eternal object considered as adding its own unique contribution to each actual occasion. This unique contribution is identical for all such occasions in respect to the fact that the object in all modes of ingression is just its identical self.*[8] *But it varies from one occasion to another in respect to the differences of its modes of ingression. Thus the metaphysical status of an eternal object is that of a possibility for an actuality. Every actual occasion is defined as to its character by how these possibilities are actualized for that occasion. Thus actualization is a selection among*

> *possibilities. More accurately, it is a selection issuing in a gradation of possibilities in respect to their realization in that occasion.* (159)

This portion of the first metaphysical principle refers to the third heading above, namely "the general principle which expresses its ingression in particular actual occasions." What is being asserted here is that the individual essence of an eternal object is nothing other than its potential to add that unique essence to any actual occasion. So redness, for example, is not something that exists in a transcendent realm of its own (as Plato seems to have held at one stage of his thinking), but is rather just the potential for the redness of actual occasions. It is, thus, a possibility for an actuality. Note, however, that Whitehead does allow for the existence of eternal objects that have never before been expressed in this universe. Without this doctrine, it becomes impossible to account for the phenomena of evolutionary emergence, as was discussed above. From this point of view evolutionary emergence is the ingression into the creative advance of a possibility, or eternal object, hitherto unactualized in this universe. The important point is that the eternal objects are, in their very essence, potentialities for actuality, and have no independent existence in a transcendent realm of their own.

Every actual occasion becomes definite as it determines its relationship to every eternal object. The relationships of an actual occasion to an eternal object differ in respect to the "grade" of its inclusion, as has already been discussed.

> *This conclusion brings us to the second metaphysical principle: An eternal object, considered as an abstract entity, cannot be divorced from its reference to other*

> *eternal objects, and from its reference to actuality generally; though it is disconnected from its actual modes of ingression into the definite actual occasions. This principle is expressed by the statement that each eternal object has a 'relational essence.' This relational essence determines how it is possible for the object to have ingression into actual occasions.* (159-60)

This second metaphysical principle concerns the second heading above, that is that an eternal object must be comprehended in terms "its general relationships to other eternal objects as apt for realization in actual occasions." This principle refers to that characteristic of eternal objects in virtue of which both natural law, on the one hand, and the process of thinking, on the other, are possible. This principle asserts that the nature of an eternal object comprises both an individual essence, and a necessary relationship to all other eternal objects. The set of relationships in which an eternal object is embedded is termed its 'relational essence.' This relational essence determines how it is possible for the object to have ingression into actual occasions in the sense that it limits which eternal objects can simultaneously coexist in any one event. For example an occasion cannot be, at the same time, at the same place, and in the same sense, both opaque and transparent. Also an occasion may be round and red, or round and blue, but it cannot be, at the same time, in the same place, and in the same sense, both round and square.

The fact that each eternal object has an individual essence is quite important. It points to the irreducibly *qualitative* dimension of our experience. A particular shade of green is an eternal object, and as such it is implicated in infinite webs of interconnections with

other eternal objects. It can be described scientifically, or it can be enjoyed poetically, we can list the things that exhibit that particular color, or correlate it with moods. But no matter how many abstractions we bring in, we can never capture in words the unique quality of that particular color. The color cannot be fully explained. This would be a commonplace, were it not for the frequency with which it is forgotten in scientific explanations. The elements in scientific equations, say, for example, the Force, Mass and Acceleration in the equation (Force = Mass x Acceleration) are often referred to as 'quantities,' and defined in terms of the mathematical relations among them. But if Force, Mass and Acceleration were mere quantities, how would it be possible to measure them in the first place? There is something that we are measuring, and that actual something that is being measured is not, in itself, a mere number.

The relational essence, or the fact that the eternal objects are ordered in a general scheme, is also important. This is the basis for the notion of natural law, or for the fact that Nature obeys, within limits, certain rules of composition. Also, because our minds can, as it were, navigate this web of relations, thinking is possible. Since the germ of mind (in the form of decisions among alternative possibilities) is inherent in all actualities, our thoughts trace out the same web of possibilities that all occasions explore in their processes of actualization. It is for this reason that reality corresponds, insofar as it does, to thought.

> *In other words: if* A *be an eternal object, then what* A *is in itself involves* A*'s status in the universe, and* A *cannot be divorced from this status. In the essence of* A *there stands a determinateness as to the relationships of* A *to other*

> *eternal objects, and an indeterminateness as to the relationships of* A *to actual occasions. Since the relationships of* A *to other eternal objects stand determinately in the essence of* A, *it follows that they are internal relations. I mean by this that these relationships are constitutive of* A*; for an entity which stands in internal relations has no being as an entity not in these relations. In other words, once with internal relations, always with internal relations. The internal relationships of* A *conjointly form its significance.* (160)

Understanding this paragraph requires an understanding of the difference between internal and external relations. Let us assume, as Whitehead does, that to be is to be in relationship.[9] Now, are *all* of the relationships that an entity has to other entities constitutive of its very being? Those relations that are a necessary part of the constitution of an entity, such that without them the entity would not be what it is, are "internal," whereas those that are not necessary in this way are "external." Some examples will help to clarify this idea. For example, California Route 1 would not be what it is if it didn't connect the various cities on the California coast. Its relationship to those cities is internal to its nature. But the cars that travel Route 1 remain what they are whether they drive on Route 1, or on any other road. Thus a road is internally related to the places that it connects, but a car is externally related to the roads on which it is traveling. A similar relation holds among abstractions. For example, and using the customary philosophical usage, "humanness" is internally related to "rationality" because (in theory at least) a human is not human unless s/he is rational. But "humanness" is externally related to "philosopherhood," since it is perfectly possible

to be human without being a philosopher.[10] Philosophers have taken various positions on the nature of these relations.

Whitehead is taking the position that each eternal object is *internally* related to all other eternal objects, and is *externally* related to all actual occasions. Let us take, for example, the eternal object that has a particular shade of red as its individual essence. This particular shade of red stands in some determinate relationship to all other shades of red, to all other colors in the visual spectrum, to the eternal object that we designate with the word "color," and, ultimately to all other eternal objects. That particular shade of red would not be what it is were it not for its participation in this network of relations.

Whitehead then says "the internal relationships of A conjointly form its significance." The word "significance" here is of the utmost importance. The modern tradition of sensory empiricism, that traces its lineage through Locke, Berkeley, and Hume, and which has been very influential in the course of modern philosophy, makes the assumption that the elements of experience are not "significant" of one another. In other words, while I have experiences of patches of red, regions of cold, and so forth, there is no intrinsic connection among them, and they do not “signify” each other. It is assumed that all order in perception is the result either of the noting of repeated patterns, or of associations among ideas somehow established in human minds. Whitehead's doctrine entirely contradicts that understanding. Whitehead maintains, first of all, that all actual occasions are necessarily embedded in a spacetime framework such that any one occasion necessarily signifies, or is significant of, other occasions. Second, by virtue of

the system of internal relations binding eternal objects one to another, eternal objects are also significant of each either. In other words, red is "significant" of the other visual colors, of "color," of "touch" and so forth. It is reasonable for me, *seeing* a color, to reach out with my hand to *feel* that which the color is characterizing because colors are, in most circumstances, significant of tactile sensations. These intrinsic connections among eternal objects are one of the key factors in reality by virtue of which thinking is relevant to actuality.

> *Again an entity cannot stand in external relations unless in its essence there stands an indeterminateness which is its patience for such external relations. The meaning of the term 'possibility' as applied to* A i*s simply that there stands in the essence of* A *a patience for relationships to actual occasions. The relationships of* A *to an actual occasion are simply how the eternal relationships of* A *to other eternal objects are graded as to the realization in that occasion.* (160)

Each eternal object stands in internal relations to all other eternal objects. And each eternal object will ingress into all actual occasions. However, the way in which an eternal object ingresses into an actual occasion says nothing whatsoever about the eternal object itself. A particular shade of red is just what is, no matter how many, or how few, occasions make it a significant part of their aesthetic synthesis. This is the patience of eternal objects for their relations to actuality, and by virtue of this patience, we term the eternal objects "possibilities."

Before going on to the next paragraph, we require a definition of the term *prehension.* While Whitehead will develop a very elaborate theory of prehension in *Process and Reality*, it is enough for the purposes of this essay to say that an actual occasion makes some other entity in its universe part of its aesthetic synthesis by "prehending" it. When an actual occasion prehends another entity, it experiences that entity, and that entity works on it as an efficient cause. The fact that the same relationship is, at once, an experience (of), and an efficient cause is just what we would expect in an ontology based on "drops of experience."

> *Thus the general principle which expresses* A*'s ingression in the particular actual occasion* a *is the indeterminateness which stands in the essence of* A *as to its ingression into* a, *and is the determinateness which stands in the essence of* a *as to the ingression of* A *into* a. *Thus the synthetic prehension, which is* a, *is the solution of the indeterminateness of* A *into the determinateness of* a. *Accordingly the relationship between* A *and* a *is external as regards* A, *and is in internal as regards* a. *Every actual occasion* a *is the solution of all modalities into actual categorical ingressions: truth and falsehood take the place of possibility. The complete ingression of* A *into* a *is expressed by all the true propositions which are about* A *and* a, *and also -- it may be -- about other things.* (160)

The eternal object *A* can ingress into an actual occasion *a* in any number of ways without affecting the essence of *A*. On the other hand the particular way in which the eternal object *A* ingresses into the actual occasion *a* is intrinsic to the very nature of that occasion. Thus eternal objects are externally related to actual occasions whereas

actual occasions are internally related to eternal objects. When Whitehead says "Every actual occasion *a* is the solution of all modalities into actual categorial ingressions" he is using the word "modality" in the sense that it has in logic of pertaining to various modes of possibility – such as "possibly," "probably," and so forth. The point is that the process through which an actual occasion comes into being resolves all modal propositions into categorical propositions, which is to say, propositions that are unequivocally true or false.

The fact that the character of an actual occasion is just what it is, so that its relation to every eternal object and, indeed, to every past actuality, is fully determinate has a most interesting implication: *it means that logical consistency is a condition for actuality*. By logical consistency I mean adherence to the law of contradiction and the law of the excluded middle. It may be *possible* for an event to be red and for it to be blue, but it will not be both red and blue in precisely the same sense (law of contradiction). Also, every proposition we can frame about an actual event will either be true or it will be false (law of the excluded middle). This feature of actuality is important in quantum mechanics, which resolves possibilities in two steps. First, it eliminates all possibilities that would, if chosen, lead to logical contradictions between the new occasion and its past. Then, in a second step, there is a decision among the logically possible outcomes that remain. Whitehead makes this two step elimination discussed in quantum mechanics a feature of his metaphysics.[11]

The determinate relatedness of the eternal object A *to every other eternal object is how* A *is systematically and by the necessity of its nature related to every*

> *other eternal object. Such relatedness represents a possibility for realization. But a relationship is a fact which concerns all the implicated relata, and cannot be isolated as if involving only one of the relata. Accordingly there is* a *general fact of systematic mutual relatedness which is inherent in the character of possibility. The realm of eternal objects is properly described as a 'realm,' because each eternal object has its status in this general systematic complex of mutual relatedness.* (160-161)

The issues involved in this paragraph have been previously discussed. It is worth saying again that the determinate relatedness of the eternal objects to each other is the basis both of natural law and of thought. We think by tracing relations among eternal objects. Note that in terms of the metaphysical position that Whitehead is developing, thinking – in the sense of noting connections among eternal objects and making decisions regarding alternative realizations – is not only involved in high grade occasions like those making up our human stream of consciousness, but it is involved in every process of actualization characterizing every event in the universe. The fact that all the events making up the universe each refer to the same set of eternal objects guarantees that our thinking will be relevant to the constitution of the real world in such a way that we can, in thought, identify natural laws in the actual world.

The next three paragraphs, which contain one of the most remarkable insights in this remarkable chapter, need some significant introduction. What Whitehead is doing here is to examine more closely the relationship between the realm of eternal objects, on the one hand, and the realm of actuality, on the other. And what he says is that to

understand the ingression of eternal objects into actual occasions, we must make reference to the "spatiotemporal relationship."

What is the spatiotemporal *relationship*? Those of us educated in modern times tend to think of spacetime as container – a big box in which things happen. Whitehead, however, is using a relational theory of spacetime. The container theory imagines spacetime as an independently existing entity, a kind of geometrically structured emptiness capable of containing other entities such as atoms and flows of energy.[12] A relational theory, on the other hand, starts from the observation that without the entities with which it is implicated, spacetime is nothing whatsoever. What spacetime brings to the entities with which it is implicated is a scheme of relationship. Spcetime allows us to specify relations such as before and after, above and below, inside and outside, and so forth. Thus spacetime is, in its essence, a scheme of relationship – hence Whitehead speaks of the "spatiotemporal relationship." Leibiz, who developed a pre-relativistic relational theory of spacetime, defined time as the order of succession among entities, and space as the order of co-presence among entities. Whitehead has adapted Leibniz idea to a post-relativistic way of thought. It is important to note that the spatiotemporal relationship in a relativistic context is also an order of causal transmission. The spatiotemporal relationship specifies where and when causal influences from one event can have effects on subsequent events. Thus, in specifying the characeristics of the actual world out of which a new occasion of experience emerges, it is not enough to know the characteristics of the occasions characterizing that past, it is also necessary to know the where and the when of those occasions. The specific

influences of the past on the present can only be specified in the context of a given scheme spatiotemporal relationships.

In *Uniformity and Contingency*[13] an essay that Whitehead wrote shortly before *Science and the Modern World*, he observes that all of our discussions of experience presuppose some particular spatiotemporal relationship, or a spatiotemporal scheme – but that the precise nature of that scheme in reality is not immediately obvious. After all, we have experiences in the spacetime of waking experience, but we also have experiences in the various spacetimes of dreams. How do we know which spacetime is the "real" one? Whitehead acknowledges that dreams have a space and time of their own which is quite separate from the space and time of waking reality. But he points out that the spacetime of waking consciousness, which he calls "the dominant spacetime continuum," exhibits itself as possessing a peculiar kind of uniformity. He says "the fitting in of distinct apprehended processes into one dominant continuum -- for example, my life in the morning with my life in the afternoon of same day – can only mean that the apprehended process of the morning has disclosed a scheme of relations amid relata, which extends beyond itself (i.e., beyond my life of the morning), so that my experience of the afternoon is nothing else than the apprehension of a process which is included in this predetermined scheme, and it is apprehended as being thus included."[14]

The language here is technical, but what is meant is something that we take entirely for granted in our lives. Each day, the Sun, the Moon, the stars and the planets do their stately dance, and every act of my waking experience can be timed by those planetary movements. Also, all of my waking acts can be unambiguously located somewhere in

proximity to the planet Earth. Each moment of my day discloses itself as part of the uniform scheme of relations among actual occasions that define earth-local time and space. But my dreams are entirely outside of that scheme. In my dreams, I have experiences that do not happen anywhere in the biosphere. Also, as Whitehead points out, since we now, post relativity, assimilate space to time, our dreams also happen at a time that also does not find a place within the planetary dance that defines waking time.[15]

Thus, if we want to claim (as Whitehead does at this point in his philosophical development) that what is truly actual is what we experience in waking life, then he can only do so by saying that what is truly actual is what falls within the scheme of relations that defines our dominant spacetime continuum.

This particular doctrine is one that Whitehead modifies in *Process and Reality*. In that work, Whitehead ceases to think of the one, dominant, spatiotemporal continuum of waking life as the decisive character of actuality. Rather, he suggests that each society defines its own spatiotemporal scheme, so that actuality is patient of many such schemes. The spatiotemporal scheme of physics is, thus, not the only spacetime of actuality. Rather it is just the spacetime in which low grade actual occasions operate in our cosmic epoch.[16]

Having said this, however, there still remain questions about the precise character of this dominant spacetime continuum. To understand these questions, we will have to

take a look at the overall history of mathematics, particularly during the past few centuries.[17]

A deep concern for mathematical truth first surfaced in the Western tradition with the Pythagoreans of Ancient Greece. Pythagoras was among the first to take significant notice of the mathematical patterning in nature, and is said to have been enormously impressed by the discovery of the mathematical relationship between musical notes and the lengths of the strings on which they are struck. Pythagoras and his followers were particularly concerned with what we now call the natural numbers (1, 2, 3 . . .), but at the time when he was working, numbers had not yet attained the level of abstraction that they have for us today. So the Pythagoreans seem to have imagined that the numbers were, at the same time, logical units of counting and regular geometrical shapes. Also, they took very seriously what we would now call "numerology," so that, for example, the number "two" was not, for them, completely distinguishable from the concept of "marriage." Given the concrete richness with which they interpreted numbers, it is not entirely surprising that they imagined the fundamental building blocks of the universe to be nothing but numbers.

Their ideas have come to seem a bit quaint to us, but nonetheless the Pythagorean school was the first to give expression to the intuition that numbers are somehow fundamental to the nature of the real.

The Greeks also made great strides in geometry, and the decisive importance of Euclid's *Elements* in the development of Western thought can hardly be exaggerated.

Both the Pythagoreans and the Greek geometers shared the conviction that mathematical intuition is a direct insight into the nature of actuality. The Pythagoreans were quite ambitious in their imaginings, thinking that the natural number series itself gave the key to all of reality, but the geometers, too, operated on the fundamental assumption that the theorems of geometry described exactly the nature of the space in which we find ourselves.

Let us pause for a moment to consider the implications of this assumption. As we moderns sometimes say, we find ourselves "thrown" into a universe the nature of which is far from obvious. We examine the world, and we discover that we have the capacity to abstract from the flow of events certain patterns that somehow transcend the events in which they are implicated. Numbers and geometrical forms stand out as early achievements of this abstracting activity as it reaches self-consciousness. Whatever pile of things we encounter, we can always, in principle at least, count it and discover the number of things of which it is comprised. The surface of the Earth consists of areas, all natural things have volume. Geometry, therefore, is relevant to all real things. It seems, then, as if our mathematical intuitions are direct insights into nature herself. The subsequent history of mathematics has shown that this is not precisely the case, and it is a look at this history that will enable us to follow Whitehead's reasoning.

The first shock to the understanding of mathematics as a direct insight into nature occurred among the Pythagoreans themselves. They made the disturbing discovery that there are numbers that are not ratios of the natural numbers. These numbers, such as π

and the $\sqrt{2}$, are what we now call irrational numbers. Any serious attempt to calculate with numbers will stumble on the irrationals very easily. For example, the ratio of the length of the side of a square to the diagonal of the same square is irrational ($\sqrt{2}$). So is the ratio of the circumference of a circle to its diameter (π). And yet, if some numbers are irrational, then those numbers are something other than the simple countable quantities that we find in nature.

A similar difficulty befell mathematicians in relation to negative numbers. These numbers became known in Europe through Arab texts, but most mathematicians of the 16th and 17th centuries did not accept them as numbers or, if they did, would not accept them as roots of equations. They were, by some mathematicians, called "absurd numbers."[18] Again, the difficulty with negative numbers is that they do not correspond to any actual thing that is found in nature. And, as if negative numbers weren't bad enough, mathematicians in the 16th century stumbled on the existence of imaginary numbers (such as $\sqrt{(-1)}$). These, like irrational numbers and negative numbers, are quite useful in computation, but correspond to nothing whatsoever that is even imaginable in the natural world.

Through all of these crises, the concept of number was broadened. The "original numbers," the natural numbers, turn out to be merely *a special case of* a *more general notion.*

A similar development took place in geometry. Since ancient times, mathematicians had been uncomfortable with Euclid's parallel axiom. While it was agreed that parallel lines

could never meet, something about that particular axiom seemed complex and inelegant. Indeed, Euclid himself seemed somewhat uneasy about it, because he first proved all the theorems he could without it, and then brought it in later when he could go no farther without it.[19] Many attempts were made to restate the axiom, or to derive it from Euclid's other axioms. None of these attempts succeeded. Finally, Gerolamo Saccheri (1667-1733), a Jesuit priest and professor at the University of Paris, thought that he might be able to prove the parallel axiom by assuming it was incorrect, and then deriving a contradiction from that assumption. When Saccheri assumed that there are no parallel lines, he was able to derive a contradiction. But when he assumed that there are at least two parallel lines, try as he might he could not derive a contradiction.[20] Gradually, mathematicians were led to the realization that "any collection of hypotheses which did not lead to contradictions offered a possible geometry."[21] Furthermore, it emerged that, assuming that the universe is sufficiently large, there is no empirical way to judge which of the possible geometries that mathematicians can articulate is actually the geometry of the space in which we live.

Again, Euclidean geometry turns out to be *a special case of* a *more general notion.* The realization that mathematical intuitions are *not* direct intuitions into the nature of the actual world came as a shock to mathematicians, and reverberations of this shock are still being felt today. Clearly, mathematical descriptions of the physical world are both applicable and immensely useful. On the other hand, both arithmetically and geometrically, mathematics *transcends* the actual world by including types of numbers and types of geometries which are not direct abstractions from the actual world itself. For our current purposes, what is most important is the realization that there are

multiple geometries of space. Thus there is no metaphysical reason why the space of our waking experience has to be Euclidean.

The relevance of this conclusion to the description of the actual world became overwhelming with Einstein's theory of relativity. The Special Theory of Relativity not only treats space and time together as a four dimensional continuum, but by introducing the relativity of simultaneity, it breaks spacetime up so that any entity with a given spacetime trajectory experiences, in effect, its own unique space and time. Here we are so far from Euclidean space that the situation becomes impossible to visualize. In the General Theory Of Relativity, this situation is further complicated, not only by describing spacetime in terms of non-Euclidean geometry, but also by the introduction of the idea that spacetime itself is "curved," or twisted by the existence of bodies within it.

The theory of relativity, too, is not above question. While the Special Theory has stood up to empirical testing so far, it remains an empirical theory, subject to disconfirmation. Whitehead himself wrote a book in which he rejected, on solid philosophical grounds, the General Theory of Relativity, and re-worked Einstein's equations so that he could regard space as basically Euclidean and treat gravity as a force.[22]

The general conclusion is that we do not know which geometry applies to the spacetime of our actual world, and that there are no *a priori* grounds on which we can decide the question.

On the other hand, while we do not know the precise geometry of the spacetime of actuality, we do know that actual occasions are never found in isolation. Rather they grow out of a past which was, itself, constituted by other occasions of experience, they are simultaneous with an indefinite number of actual occasions that are also coming into being, and they intrinsically refer to a future which will be actualized by occasions of experience that have not yet taken place. Thus all actual occasions, even those that take place in dreams, are situated in a spacetime continuum of some sort.

With this background, we can return to our examination of the text.

> *In respect to the ingression of* A *into an actual occasion* a, *the mutual relationships of* A *to other eternal objects, as thus graded in realization, require for their expression a reference to the status of* A *and of the other eternal objects in the spatio-temporal relationship. Also this status is not expressible (for this purpose) without a reference to the status of* a *and of other actual occasions in the same spatio-temporal relationship.* (161)

By virtue of their internal interconnections, the eternal objects form a 'realm' of possibilities. The eternal objects are, in general, patient of ingression into actuality, all actuality is spatiotemporal, and so each eternal objects has a status in relation to the spatiotemporal relationship itself. This is obvious for eternal objects such as "near" and "far." But it also applies, to simple eternal objects such as "red" which cannot, in the same sense, co-exist with "green" at the same spatiotermporal position, but which *can* coexist with round and bouncy.

In their ingression into specific actual occasions, each of which defines a unique spatiotemporal location, the eternal objects which come to positively characterize the a given occasion must be:

- Logically consistent among themselves
- Logically consistent *with the eternal objects characterizing the spatiotemporally arrayed occasions of the past out of which they arise.*

Thus we cannot understand the ingressions of eternal objects into actual occasions without reference to the particular spatiotemporal relationships which characterize actuality.

> *Accordingly the spatio-temporal relationship, in terms of which the actual course of events is to be expressed, is nothing else than a selective limitation within the general systematic relationships among eternal objects. By 'limitation,' as applied to the spatio-temporal continuum, I mean those matter of fact determinations – such as the three dimensions of space, and the four dimensions of the spatio-temporal continuum – which are inherent in the actual course of events, but which present themselves as arbitrary in respect to* a *more abstract possibility. The consideration of these general limitations at the base of actual things as distinct from the limitations peculiar to each actual occasion, will be more fully resumed in the chapter on 'God.'* (161)

The idea of spacetime as a limitation on possibility needs some expansion. First, we know that every actual event takes place at a position in spacetime. The law of contradiction and the law of the excluded middle only come into effect in relation to an actual spatiotemporal position. Second, consider the propagation of electromagnetic radiation. We know that the electromagnetic radiation from a source can only have efficient causal effects within a sphere around the source that expands at the speed of light. So, if we are too far away in spacetime, we cannot have a direct experience of that particular radiation. It is *impossible* for the radiation to be felt by those who *are too far away*. Here we can clearly see spacetime as a limitation on possibility. Examples of this second sort can be multiplied indefinitely, and are at the core of the physical description of actuality. Also, consider the causal relations involved in viewing an object outside of our bodies. We always see objects from a perspective – which is to say that only those causal relations implicating that region of the object which is "turned our way" are directly relevant to our perceptual experience. The perspective under which we view objects is also a characteristic of the spacetime relationship. Spacetime, rather than merely containing eternally formed entities, is now the spacetime relationship, a factor in actuality which channels, or limits, the causal interactions among occasions, and so plays a role in establishing the initial conditions (but not the utlimate character) of a new occasion.

The fact that we live in an actual world means that actual worlds are possible. Also, an actual world is the creative advance of a *society* of actual occasions, and some form of spatiotemporal scheme of relations is presupposed by the notion of social interactions among occasions. In terms of a scheme of spatiotemporal relations, actual occasions can

point to, or indicate each other, and can anticipate where and when future occasions may occur. But any scheme of spatiotemporal relations is internal to the society which uses it. We cannot specify "where" or "when" the whole society is, except in the framework of some larger society; and we have no empirical or metaphysical grounds on which we can assert the ultimacy of any society, no matter how large. The cosmic epoch in which we are situated, and about which we learn through astrophysics, may be only one of an infinite number of such epochs. Our knowledge can put no limits on the "space" of possibility which houses all actualities. Thus spacetime might have been otherwise.

Let us imagine, as we did in modern times, before the Theory Of Relativity, that space is an infinite three dimensional cubical grid, and that time is a linear dimension somehow perpendicular to the other three. Now we can confidently indicate the occasions around us, we can meaningafully analyze causal relations among them, and we can predict possible locations for new occasions, in terms of four coordinates relative to the four axes (in respect to some arbitrary "origin") of spacetime. This feature of actuality cannot be deduced from the mere patterns of interconnections among eternal objects. Those patterns would equally apply if space had, let us say, four dimensions so that the whole spatiotemporal continuum were then five dimensional. In that case, actual occasions could take place in regions which are entirely undefined in our current scheme of four dimensional reality.

The general interconnections in the realm of eternal objects constitute the outer limits of what is possible. The geometry of spacetime is a limitation on that freedom. Empirically, we do not actually know, in any certain way, the geometry of our dominant

continuum. And it is not difficult to imagine other actualities that might operate in spacetimes very different from the one that characterizes our cosmic epoch. Given the vastness of the realm of possibility, the question as to why it is that our spacetime geometry is as it is comes forcefully to our attention. Whitehead assigns responsibility for ultimate decisions of this kind to the entity that he calls "God."

> *Further, in any particular consideration of a possibility we may conceive this continuum to be transcended. But in so far as there is any definite reference to actuality, the definite* how *of transcendence of that spatio-temporal continuum is required. Thus primarily the spatio-temporal continuum is a locus of relational possibility, selected from the more general realm of systematic relationship. This limited locus of relational possibility expresses one limitation of possibility inherent in the general system of the process of realization. What ever possibility is generally coherent with that system falls within this limitation. Also what ever is abstractedly possible in relation to the general course of events – as distinct from the particular limitations introduced by particular occasions – pervades the spatio-temporal continuum in every alternative spatial situation and at all alternative times.*
>
> *Fundamentally, the spatio-temporal continuum is the general system of relatedness of all possibilities, in so far as that system is limited by its relevance to the general fact of actuality.* (161-62)

This paragraph highlights a distinction which is sometimes blurred in everyday discourse. People are exploring the outcome of a given situation, they often say

"anything is possible." But it is never the case that "anything is possible." First of all, as we have seen, possibility is structured. It is, for example, not possible for the same thing to be round and square at the same time, in the same place, and in the same sense. So even abstract possibility is limited. This paragraph points to a further limitation on possibility -- a limitation which is imposed by the past. No concrescence can come to be characterized by an eternal object that is not logically consistent with the actual world out of which it has arisen. It is *abstractly* possible that I could have wings, but it would not be consistent with my past to discover that I suddenly have wings right now. The actual world of any new concrescence is characterized by an order of succession (time), and an order of co-presence (space). What is *actually* possible for me in this moment can only be specified as a function of all of those occasions that characterize my spatiotemporal past, and by the specific spatiotemporal relations that obtained among them. Thus, "the status of all possibility in reference to actuality requires a reference to this spatio-temporal continuum."

As we have seen, the spacetime geometry characterizing our universe might, logically, have been different. The limitation of actuality to a given spacetime geometry "expresses one limitation of possibility inherent in the general system of the process of realization." Of course, if we ignore the limitations imposed by the particularities of the past, then any relational possibility that is consistent with the particular geometry of spacetime is possible in all spaces and at all times.

> *Also it is inherent in the nature of possibility that it should include this relevance to actuality. For possibility is that in which there stands achievability, abstracted from achievement.* (162)

Whitehead is here emphasizing that the eternal objects do not exist in an isolated, transcendent realm of their own. Rather, some reference to the possibility of actualization is inherent in their natures.

These few paragraphs suggest an understanding of spacetime which is very different from our more ordinary notion of spacetime as a container in which things exist and move. Spacetime is here defined as a relationship among actualities that channels causal interactions, and thus defines the scope of possibilities that can be realized by subsequent actualities.

In work that I have done in other contexts,[23] I have been suggesting that the "transphysical realms," i.e., the places in which we dream, have lucid dreams, have out of body experiences and, quite possibly, the places in which we find ourselves after the death of our physical bodies, are like the physical world in that they consist of causally interacting actual occasions of experience, but are unlike the physical world in that (among other things) the geometries of their dominant continua are different. The analysis of spacetime which Whitehead offers here supports those ideas.

> *It has already been emphasized that an actual occasion is to be conceived as a limitation; and that this process of a limitation can be still further characterized as a gradation. This characteristic of an actual occasion (*a, *say) requires further elucidation: An indeterminateness stands in the essence of any eternal object (*A, *say). The actual occasion* a *synthesizes in itself every eternal*

> *object; and, in so doing, it includes the* complete *determinate relatedness of* A *to every other eternal object, or set of eternal objects. This synthesis is a limitation of realization but* not *of content. Each relationship preserves its inherent self identity. But grades of entry into this synthesis are inherent in each actual occasion, such as* a. *These grades can be expressed only as relevance of value. This relevance of value varies – as comparing different occasions – in grade from the inclusion of the individual essence of* A *as an element in the aesthetic synthesis (in some grade of inclusion) to the lowest grades which is the exclusion of the individual essence of* A *as an element in the aesthetic synthesis. In so far as it stands in this lowest grade, every determinate relationship of* A *is merely ingredient in the occasion in respect to the determinate* how *this relationship is an unfulfilled alternative, not contributing any aesthetic value, except as forming an element in the systematic substratum of unfulfilled content. In a higher grade, it may remain unfulfilled but be aesthetically relevant.* (162)

The relationships among eternal objects are internal to the eternal objects themselves. Thus, the eternal objects cannot be separated from their interconnections and must bring all of their interconnections with them into each actual occasion. In this way the entire realm of eternal objects is relevant to each actual occasion. However, actual occasions may choose the extent to which they *value* each eternal object and each of its possible interconnections. Those that they value highly figure prominently in the aesthetic synthesis which the occasion achieves, and can thus be said to *characterize* that actual occasion. Whitehead calls this "a high grade of inclusion." Other grades of inclusion include essential exclusion from the aesthetic synthesis (what Whitehead will

later call "negative prehension), or partial inclusion as a minor character, or as a possibility considered but not chosen.

> *Thus* A, *conceived merely in respect to its relationships to other eternal objects, is '*A *conceived as* not-being'*; where ' not-being' means 'abstracted from the determinate fact of inclusions in, and exclusions from, actual events.' Also '*A *as* not-being *in respect to a definite occasion* a*' means that* A *in all its determinate relationships is excluded from* a. *Again '*A *as being in respect to* a*' means that* A *in some of its determinate relationships is included in* a. *But there can be no occasion which includes* A *in all its determinate relationships; for some of these relationships are contraries. Thus, in regard to excluded relationships,* A *will be* not-being *in* a, *even when in regard to other relationships* A *will be* being *in* a. *In this sense, every occasion is a synthesis of* being *and* not-being. *Furthermore, though some eternal objects are synthesized in an occasion* a *merely* qua not-being, *each eternal object which is synthesized* qua being *is also synthesized* qua not-being. 'Being' *here means 'individually effective in the aesthetic synthesis.' Also the 'aesthetic synthesis' is the 'experient synthesis' viewed as self-creative, under the limitations laid upon it by its internal relatedness to all other actual occasions. We thus conclude – what has already been stated above – that the general fact of the synthetic prehension of all eternal objects into every occasion wears the double aspect of the indeterminate relatedness of each eternal object to occasions generally, and of its determinate relatedness to each particular occasion. This statement summarizes the account of how external relations are possible. But the account depends upon disengaging*

> *the spatio-temporal continuum from its mere implication in actual occasions – according to the usual explanation – and upon exhibiting it in its origin from the general nature of abstract possibility, as limited by the general character of the actual course of events.* (162-63)

At the time that Whitehead was writing *Science and the Modern World*, Josiah Royce and other British and American philosophers were actively engaged in a defense of Absolute Idealism. One of their arguments was that an entity would not be precisely what it is if *any* of the relations in which it is involved were different from precisely what they happen to be. Thus all relations must be internal, the universe is already, always and forever just what it is, all connections among things (which must be free from any randomness or freedom) are rational through and through, and the universe is pure, immutable being. Whitehead, with his more empirical temperament, wanted to make room in his philosophy for real process, for the experience of freedom, and for adventure in a creative advance towards novel forms of beauty.

Whitehead defines the not-being of an eternal object two ways:

- First, a particular eternal object is not-being when considered in terms only of its relations to other eternal objects, "abstracted from the determinate fact of inclusions in, and exclusions from, actual events." It seems that in this context, Whitehead is using the term "being" in a way that is synonymous with his use of "actuality."
- Second, in respect to a particular occasion (*a*), *A* as not-being means either:
    - *A* as excluded from *a* in terms of *all* relations to other eternal objects
    - Or *A* insofar as *some* of its relations are excluded from *a*.

The being of an eternal object (*A*) in respect of an actual occasion (*a*) is the inclusion of some of *A*'s relational essence in *a*'s aesthetic synthesis. Thus he can say that an actual occasion is a synthesis of being and not-being. Not-being in respect of an actual occasion is, as Whitehead has defined it, external relation to that occasion. Being in respect to an actual occasion is internal relation to that occasion. Thus, Whitehead circumvents the arguments of the Absolute Idealists and maintains the existence of external, as well of internal, relations.

To summarize, eternal objects are internally related among themselves, actual occasions are internally related to the actual occasions constituting the actual world out of which they arise. Actual occasions are externally related to eternal objects (except insofar as those eternal objects have ingression in the past world out of which they arise), and externally related to the occasions of their future worlds.

The last sentence of this paragraph reads. "But the account depends upon disengaging the spatiotemporal continuum from its mere implication in actual occasions -- according to the usual explanation -- and upon exhibiting it in its origin from the general nature of abstract possibility, as limited by the general character of the actual course of events." Before we can unpack this sentence, there are two issues which we need to explore. The first is "What is the relationship of possibility to actuality?" That issue was mentioned briefly earlier in this commentary, but we return to it here in a more thorough way.

I take it that we can all use the words "potentiality" and "actuality" so that we generally know what they mean. But what, precisely, is the difference between a potentiality and

an actuality? We might say, "well, an actuality is *real*, and a potentiality is not." But this answer only serves to obscure the issue. What, after all, do we mean by "real?"

We must first understand that for Whitehead (as for all of those interpretations of quantum mechanics that are influenced by Heisenberg) potentiality and actuality are two different species of real things. It is very difficult to interpret quantum mechanics without some notion of real potentials. Indeed, even common sense discriminates between real potentials (e.g., "I can get out my chair and leave my office via the door") and unreal potentials (e.g., "I can sprout wings and fly out of my window"). So the distinction between the potential and the actual is not the same as the distinction between the unreal and real. But, clearly, the potential and the actual are *different*. So what is the difference?

Whitehead suggests that there are two essential differences between the potential and the actual.

- First, the potential is, in its nature, broader than the actual. In any given situation, there are multiple potentials, but only one, non-contradictory set of them can be realized. Thus, the process of actualization is understood as a process of *decision*, or "cutting off," in which all but one particular set of logically consistent potentials is discarded. Actualization occurs when this decision process is completed.
- Second, since actual occasions cannot exist except as an aesthetic synthesis of the past out of which they grow, actual occasions are inherently social, and social relations are only possible in a situation in which entities share a spatiotemporal

> relation among themselves. Spacetime *is* the spatiotemporal relationship among actual occasions.

Thus the possible becomes actual through a process of decision by a member of a socially (and, therefore, spatiotemporally) embedded actual occasion. Decision and spacetime are the key factors that allow the potential to actualize.

Whitehead, then, is saying something about spacetime in its role as a precondition for actualization. To understand what he is saying, we must now ask as to the "usual explanation" for the spatiotemporal continuum. I am assuming that the "usual explanation" is the container theory of space that is common in post-Newtonian scientific thought. In that way of thinking, the primary function of space is to differentiate the various entities by giving each one a separate position. each entity in such a space is just what is (for example, a point-particle of mass). It has the character that it has no matter where or when it is. This is what Whitehead refers to in Chapter Three of *Science and the Modern World* as "the fallacy of simple location." In this mode of thinking, the nature of the entity is eternally fixed, is independent of its position, and thus is not a function of causal influences from the past. Because the character of the entities that ultimately constitute actuality are fixed, and because the relations among the eternal objects are also fixed, the entire relam of eternal objects is internally (rather than externally) related to these entities. The fact that the entire realm of eternal objects is internally related to the entities making up the actual world means that those entities are not processes, and have no freedom. Thus their actions should be completely determined. In this way we end up back in the block universe of the absolute idealists.

But in the context of a process metaphysics, space functions not only to separate entities, but also to connect them. Space limits possibility by conditioning causal transmission. With this understanding of the spacetime relationship, we are in a universe of process. Spacetime and events are seen as coherently related one to the other, and the external relationship of the realm of possibilities to actualities in general is established.

It is the externality of the relationship between the realm of eternal objects and the emerging actualities which allows for both freedom and order in the universe of our experience.

> *The difficulty which arises in respect to internal relations is to explain how any particular truth is possible. In so far as there are internal relations, everything must depend upon everything else. But if this be the case, we cannot know about anything till we equally know everything else. Apparently, therefore, we are under the necessity of saying everything at once. This supposed necessity is palpably untrue. Accordingly it is incumbent on us to explain how there can be internal relations, seeing that we admit finite truths.* (163)

This paragraph introduces an examination of the structure of possibility. Whitehead has maintained that the eternal objects are internally connected, which means that no eternal object is what it is apart from its relations to other eternal objects. Whitehead has already softened this claim with the notion that each eternal object has an 'individual essence' as well as a 'relational essence.' But the question here is something

like: "In this infinite network of internal interconnections, how is it possible to meaningfully isolate the groups of eternal objects with which we work in our finite processes of thought?"

> *Since actual occasions are selections from the realm of possibilities and thus, finite truths, the ultimate explanation of how actual occasions have the general character which they do have, must lie in an analysis of the general character of the realm of possibility.* (163)

This analysis of the structure of possibility is also important in relation to the creative advance of actuality. Since each actual occasion embodies, in its aesthetic synthesis, some subset of the network of relations internally connecting the eternal objects, how are actual occasions themselves possible?

> *The* analytical character *of the realm of eternal objects is the primary metaphysical truth concerning it. By this character it is meant that the status of any eternal object* A *in this realm is capable of analysis into an indefinite number of subordinate relationships of limited scope. For example if* B *and* C *are two other eternal objects, then there is some perfectly definite relationship* R(A, B, C) *which involves* A, B, C *only, as to require the mention of no other definite eternal objects in the capacity of relata. Of course, the relationship* R(A, B, C) *may involve subordinate relationships which are themselves eternal objects, and* R(A, B, C) *is also itself an eternal object. Also there will be other relationships which in the same sense involve only* A, B, C. *We have now to examine how, having*

> *regard to the internal relatedness of eternal objects, this limited relationship* R(A, B,C) *is possible.* (163-64)

The point here is that we can, in a meaningful way, perform algebra-like operations with all eternal objects. The previous paragraph will seem clearer if we express it in terms of a particular example. Let us assume that A is the eternal object that we designate by the word "color." Color has its individual essence, and its relational essence. The relational essence of "color" connects it to "shape" and "size." That is, we can allow *A* to stand for color, *B* to stand for shape, and *C* to stand for size, and R to stand for the relation "together in a visual object." Thus *R(A, B, C)* will stand for a visual object characterized by a color, a shape and a size. It is apparent that we can understand this relationship without referring to the relationship between color and texture, or between size and weight, and so forth. It is important to note:

- That this relationship may also involve subordinate relationships, such as *S(B, C)* which, let us say, stands for the togetherness of shape and size in a visual object;
- That there will be other relationships between A, B and C, for example *T(A, B, C)* which designates color, shape and size as elements contributing the style of an article of clothing; and
- That that all relations (including, of course, R, S, and T) are, themselves, eternal objects.

The fact that the realm of eternal objects can be factored in this way is its "analytical character." Note that if the realm of eternal objects did not have this character, it would be impossible for finite thought to gain a foothold in it.

The reason for the existence of finite relationships in the realm of eternal objects is that relationships of these objects among themselves are entirely unselective, and are systematically complete. We are discussing possibility; so that every relationship which is possible is thereby in the realm of possibility. Every such relationship of each eternal object is founded upon the perfectly definite status of that object as a relatum in the general scheme of relationships. This definite status is what I have termed the 'relational essence' of the object. This relational essence is determinable by reference to that object alone, and does not require reference to any other objects, except those which are specifically involved in its individual essence when that essence is complex (as will be explained immediately). The meaning of the words 'any' and 'some' springs from this principle -- that is to say, the meaning of the 'variable' in logic. The whole principle is that a particular determination can be made of the how *of some definite relationship of a definite eternal object* A *to a definite finite number n of other eternal objects,* without *any determination of the other n objects,* X_1, X_2, . . . X_n, *except that they have, each of them, the requisite status to play their respective parts in that multiple relationship. This principle depends on the fact that the relational essence of an eternal object is not unique to that object. The mere relational essence of the each eternal object determines the complete uniform scheme of relational essences, since each object stands internally in all its possible relationships. Thus the realm of possibility provides a uniform scheme of relationships among finite sets of eternal objects; and all eternal objects stand in all such relationships, so far as the status of each permits.* (164)

This paragraph is extremely dense. It makes the following points:

- Because we are discussing only the realm of possibility, no connections among the eternal objects are necessary connections. The eternal objects are possibilities, and the relations among them are merely possible relations among possibilities. In this sense, the relations are 'entirely unselective.'
- All relations which are possible are in the realm of possibility – which is to say that there is an eternal object for each relational possibility. In this sense the realm of eternal objects is 'complete.'
- Every relation which an eternal object has with any other eternal object is part of its relational essence, but the specification of the relational essence does not require reference to other, specific eternal objects (except when the eternal object itself is complex, in a sense to be explained shortly). The specification of the relational essence does, however, require a reference to *classes* of other objects. For example, we can discuss the relational essence of "visual object" without a specific reference to any color, any shape, or any size. But, referring back to our earlier example, the relation R(A, B, C) is only relevant when A, B, and C have 'the requisite status to play their respective parts in that multiple relationship' – i.e., in this case, A must belong to the class of colors, B must belong to the class of shapes, and C must belong to the class of sizes.
- Because we can reference classes of eternal objects in this way, we can make sense of 'any' and 'some,' and thus of the concept of a *variable* in logic. A variable is always a reference to a class of objects.

- All relations are eternal objects, and all individual eternal objects stand in all relations to which they are relevant.
- The relational essence of each eternal object includes all possible relations between it and all other eternal objects, including its relations to all possible relations. Thus, there is a uniform scheme of relations and all eternal objects stand in all such relationships insofar as they are relevant to each.

> *Accordingly the relationships (as in possibility) do not involve the individual essences of the eternal objects; they involve* any *eternal objects as relata, subject to the proviso that these relata have the requisite relational essences. [It is this proviso which, automatically and by the nature of the case, limits the 'any' of the phrase 'any eternal objects.'] This principle is the principle of the* Isolation Of Eternal Objects *in the realm of possibility. The eternal objects are isolated, because their relationships as possibilities are expressible without reference to their respective individual essences.* (insert in original) (165)

The relational essences of eternal objects do not belong to them in virtue of their individual essences, but rather belong to them in virtue of the classes to which they belong. For example, the relation of a particular shade of red to a particular shape and size is not a function of the particular shade of red, the particular shape and the particular size, but rather belongs to all three by virtue of the fact that they are 'a color,' 'a shape,' and 'a size.' In this way, the individual essences are "Isolated." The relational

essences of eternal objects are relations among classes, and do not depend on the specifics of individual essences at all.

> *In contrast to the realm of possibility the inclusion of eternal objects within an actual occasion means that in respect to some of their possible relationships there is a togetherness of their individual essences.* (165)

On the other hand, when eternal objects are included in actual occasions, it is their individual essences (as they appear in some particular relation) that are brought together. There is, thus, a contrast between the 'isolation' of eternal objects in the realm of possibility and the 'togetherness' of eternal objects in actual occasions.

> *This realized togetherness is the achievement of an emergent value defined -- or, shaped -- by the definite eternal relatedness in respect to which the real togetherness is achieved. Thus the eternal relatedness is the form -- ειδος --; the emergent actual occasion is the superject of informed value; value, as abstracted from any particular superject, is the abstract matter – the ϋλη -- which is common to all actual occasions and the synthetic activity which prehends valueless possibility in this superjicient informed value is the substantial activity. This substantial activity is that which is omitted in any analysis of the static factors in the metaphysical situation. The analyzed elements of the situation are the attributes of the substantial activity.* (165)

This paragraph concerns the relationship of actuality to possibility. Before we can discuss this possibility, we need to step back and look at the relationship between form (ειδος) and substance (ϋλη). Given that the words are Aristotelian, and are here spelled in Greek characters, we can assume that what is being referred to is the way in which these terms were employed by Aristotle.

Aristotle's understanding of form is quite similar to Whitehead's understanding of eternal objects, and is, by now, familiar to us. But his understanding of substance is quite different from the modern understanding of a substance as a separate, self-existing bit of "stuff." For Aristotle, the *hylē*, or substance, is the 'that out of which' a finite being is made. So, for example, we can return to our child's ball, and observe that that out of which it is made is red rubber. So, in this case, "red rubber" is the substance (*hylē*) and, and "ball" is the form. But red rubber, itself, is subject to a similar analysis. Just as the child's ball involves the formative activity of the idea of "ball" onto the red rubber substance, so there is some substance on which the formative activity of "redness" and "rubberness" has acted to give the 'that out of which' the ball was made. Aristotle follows this regress until he arrives a formless potentiality for the imposition of form, which is the ultimate *hylē* in his metaphysical system.

By analogy to Aristotle, Whitehead sees each actual occasion, or each finite existence, as the coming together of substance and form. In his case, the form, or idea, consists of those particular eternal objects which achieve a togetherness of individual essences in the aesthetic synthesis of a particular actual occasion. But the substance is here identified as *value*, rather than as "stuff," and the activity which draws together the individual

essences of eternal objects (which are valueless in themselves) into an uniquely valuable aesthetic synthesis is the activity of value in shaping actuality.

To understand the reference to "superject" and "superjicient" (Whitehead later uses the term "superjected," rather than the cumersome "superjecient") we recall that the ultimate reality to which Whitehead is pointing is composed of causally interacting drops of experience – i.e. each event in the universe *is,* on the inside, the experience of a subject. As the experience of the momentary subject comes to completion in definiteness, the aesthetic synthesis is *superjected*, or projected out into the future for the prehension of later occasions. So causally interacting drops of experience are referred to as "subject-superjects."

What Whitehead is here calling 'value' is, in *Process and Reality*, referred to as 'Creativity." Creativity is the 'category of the ultimate,' and all of the other factors discriminated in the metaphysical system are attributes of creativity.

> *The difficulty inherent in the concept of finite internal relations among eternal objects is thus evaded by two metaphysical principles, (i) that that the relationships of any eternal object A, considered as constitutive of A, merely involve other eternal objects as bare relata without reference to their individual essences, and (ii) that the divisibility of the general relationship of* A *into a multiplicity of finite relationships of* A *stands therefore in the essence of that eternal object. The second principle obviously depends upon the first. To understand* A *is to understand the* how *of the general scheme of relationship. This scheme of relationship does not require the individual uniqueness of the*

> *other relata for its comprehension. This scheme also discloses itself as being analyzable into a multiplicity of limited relationships which have their own individuality and yet at the same time presupposes the total relationship within possibility.* (165)

This passage summarizes what has so far been said in respect to the analytic character of the realm of eternal objects.

> *In respect to actuality there is first the general limitation of relationships, which reduces the general unlimited scheme to the four-dimensional spatio-temporal scheme. This spatio-temporal scheme is, so to speak, the greatest common measure of the schemes of relationship (as limited by actuality) inherent in all the eternal objects. By this it is meant that,* how *select relationships of an eternal object (*A*) are realized in any actual occasion, is always explicable by expressing the status of* A *in respect to this spatiotemporal scheme, and by expressing in this scheme the relationship of the actual occasion to other actual occasions.* (165-66)

This passage summarizes what was said, earlier, about the relationship of the spacetime to the realm of eternal objects. Note (this was mentioned previously) that in *Science and the Modern World*, Whitehead is still assuming that the four-dimensional spatiotemporal scheme characteristic of our cosmic eqoch is common to all actuality, whereas in *Process and Reality*, Whitehead recognizes that there may be many epochs, each with its own characteristic spatiotemporal scheme.

> *A definite finite relationship involving the definite eternal objects of a limited set of such objects is in itself an eternal object: it is those eternal objects as in that relationship. I will call such an eternal object 'complex.' The eternal objects which are the relata of a complex eternal object will be called the 'components' of that eternal object. Also if any of these relata are themselves complex, their components will be called 'derivative components' of the original complex object. Also the components of derivative components will also be called derivative components of the original object. Thus the complexity of an eternal object means its analyzability into a relationship of component eternal objects. Also the analysis of the general scheme of relatedness of eternal objects means its exhibition as a multiplicity of complex eternal objects. An eternal object, such as a definite shade of green, which cannot be analyzed into a relationship of components, will be called 'simple.'* (166)

Here we have the definition of 'complex eternal objects.' The eternal object designating a relationship among other eternal objects is complex. We might take as a complex eternal object "rose." This eternal object cannot be understood apart from a reference to petals, fragrance, and a host of other eternal objects. An eternal object is complex to the extent that its individual essence requires, for its expression, reference to other eternal objects. Complex eternal objects may, themselves, be composed of complex eternal objects. Only those eternal objects, such as a 'definite shade of green,' which cannot be analyzed into a relationship of components will be called 'simple.'

Whitehead will, in the next few paragraphs, detail this idea, and then will make use of it a further discussion of the relationship between possibility and actuality.

> *We can now explain how the analytical character of the realm of eternal objects allows of an analysis of that realm into grades.* (166)

Note that Whitehead uses the term "grade" in two different ways in this chapter. In the earlier portion of this chapter, he speaks of eternal objects having ingression into actual occasions in a variety of "grades of relevance to the aesthetic synthesis." Now, he is using the term "grade" to mean "grade of complexity."

> *In the lowest grade of eternal objects are to be placed those objects whose individual essences are simple. This is the grade of zero complexity. Next consider any set of such objects, finite or infinite as to the number of its members. For example, consider the set of three eternal objects* A, B, C, of *which none is complex. Let us write* R(A, B, C,) *for some definite possible relatedness of* A, B, C. *To take a simple example,* A, B, C *may be three definite colours with the spatio-temporal relatedness to each other of the three faces of a regular tetrahedron, anywhere at any time. Then* R(A, B, C) *is another eternal object of the lowest complex grade. Analogously there are eternal objects of successively higher grades. In respect to any complex eternal object,* $S(D_1, D_2, \ldots D_n)$, *the eternal objects* $D_1 \ldots D_n$, *whose individual essences are constitutive of the individual essence of* $S(D_1, \ldots D_n)$, *are called the components of* $S(D_1, \ldots D_n)$. *Its obvious that the grade of complexity to be ascribed to* $S(D_1, \ldots D_n)$, *is to be*

> *taken as one above the highest grade of complexity to be found among its components.* (166-67)

Simple eternal objects, such as a particular shade of green or a particular flavor, which cannot be analyzed into component eternal objects, are of grade 0. Any eternal object which involves relations among such simple objects is of grade 1, the lowest grade of complex eternal objects. Examples of such complex objects of grade 1 include:

- R(A, B, C)
- S(A, B)
- T(A, B, C, D, E, F)

It is assumed here that A, B , C, D, E, and F are all simple eternal objects. Note that the number of eternal objects involved in the relation does not determine the grade, rather it is the highest grade of any *one* of the component eternal objects.

Examples of eternal objects of Grade 2 include:

- U((R(A, B, C), E)
- V(S(A, B), R(A, B, C))

In "U((R(A, B, C), E)," "R(A, B, C)" is an eternal object of grade 1. "U" is a relationship between "E" (an eternal object of grade 0) and "R", an eternal object of grade 1. Since one of the components of "U" is of grade 1, "U" is of grade 2. In "V(S(A, B), R(A, B, C))." Both "S" and "R" are of grade 1 so, again, "V" is of grade 2.

> *There is thus an analysis of the realm of possibility into simple eternal objects, and into various grades of complex eternal objects. A complex eternal*

> *object is an abstract situation. There is a double sense of 'abstraction,' in regard to the abstraction of* definite *eternal objects, i.e., non-mathematical abstraction. There is abstraction from actuality an abstraction from possibility. For example,* A *and* R(A, B, C) *are both abstractions from the realm of possibility. Note that* A *must mean* A *in all its possible relationships, and among them* R(A, B, C). *Also* R(A, B, C) *means* R(A, B, C*) in all its relationships. But this meaning of* R(A, B, C) *excludes other relationships into which* A *can enter. Hence* A *as in* R(A, B, C) *is more abstract than* A simpliciter. *Thus as we pass from the grade of simple eternal objects to higher and higher grades of complexity, we are indulging in a higher grades of abstraction from the realm of possibility.* (167)

The distinction being introduced here (it will be further discussed a few paragraphs down) is a crucial one. There is abstraction from possibility and abstraction from actuality. How do they differ?

Before we can answer this question, we need to pause and to look at Whitehead's use of the terms "abstract" and "concrete." The word "concrete" has come, in everyday American usage, to take on the connotations that are derived from the substance "concrete." It tends to mean something like "existing in a material or a physical form," or even (as in "give me a concrete example") "specific and particular." Whitehead, however, always uses "concrete" as the opposite of "abstract." Thus, for him, the word "concrete" means something like "whole, entire, divisible but undivided" The "abstract" is always an abstraction from the "concrete."

The realm of possibility, in its full concreteness, consists in all the eternal objects, with all of their individual and relational essences. Any reference to any particular eternal object, say A, is an abstraction form that, While A must be construed as referring to A as in all of its relations, the relational essence of A is only part of the relational essence of the entire realm. Now, if we refer to an object with a higher grade of complexity, say R(A, B, C), then this includes only a subset of A's relational essence (as well as a subset of B and C's relational essences), and so is more limited, more specific, more abstract than A simpliciter. The point is that, in the realm of possibility, the more complex the object the narrower, or more abstract, are the possibilities that it presents.

This is clear if we think about mathematics and logic. It has, in the 20th century, been demonstrated that all of higher mathematics can (assuming the laws of logic) be derived from arithmetic. Arithmetic involves numbers and one simple operation, addition (subtraction, multiplication and division can all be accomplished by suitable operations of addition). Giuseppe Peano (1858 –1932) was able to derive all of arithmetic from three primitive elements and five axioms. Thus, since all of mathematics follows from arithmetic, all of mathematics follows from just three definitions and five axioms.[24] Each of the basic elements (0, number, successor) is of grade zero. One of the axioms is of grade 1("zero is a number"). The other four are of grade 2 ("the successor of any number is a number," "0 is not the successor of any number," "no two numbers have the same successor," "any property which belongs to 0, and also to the successor of every number which has the property, belongs to all numbers")[25]. Thus these very simple eternal objects imply all of mathematics. Complex mathematical expressions are more specific, more constrained, and more abstract, than are (correctly chosen) simple ones.

It is for this reason that the logical formulations of mathematics (as, for example, in Whitehead and Russell's *Principia Mathematica*) are important, even if Gödel demonstrated that they are, if consistent, incomplete.

I would like to observe, as Whitehead would have been unlikely to do given the period in which he was writing, that *all algorithms are eternal objects*. Because of the way in which Whitehead speaks, it is tempting to imagine all of the eternal objects, simple and complex, as static. "A particular shade of red," for instance, doesn't *do* anything except for being red. Also, "three definite colors with the spatiotemporal relatedness to each other of the three faces of a regular tetrahedron, anywhere at any time" connotes an entirely static function. However, the application of complex eternal objects in the form of the algorithms with which we program computers has sensitized us to their dynamic possibilities.

Consider the algorithm by means of which a guided missile computes its spatiotemporal position. This is a complex eternal object. It seems reasonable to speculate that an actual occasion which is part of a personally ordered society of such occasions,[26] might use a similar algorithm to compute a position for its successor occasions. Something like this is necessary if we are going to construct a "process mechanics." Note, however, that the algorithms used in computers begin and end with numbers. In the case of the guided missile, there is an instrument that converts sensor data into numbers, the algorithm works on those numbers, and some process is then required to convert the new numbers back into physical actions in the guidance system. In actual occasions, however,

algorithms can operate directly on *any* eternal object. So an artist might employ a sort of algorithm which inputs the pattern of color on a canvas, and returns a particular color which might enhance the aesthetic quality by being applied at a certain position in the overall composition.

In the Goethian tradition of science, there are entities such as "the archetype of plant" which seem to remain unchanged throughout the life of the plant, and yet which produce various plant organs in response to internal and external conditions of the plant in which it is expressed.[27] It is difficult to see how these "archetypes" could be reduced to patterns of eternal objects unless those pattern could function, in a general sense, as algorithms. While Whitehead has not explored these issues, he has created a structure which can accommodate them.

> *We can now conceive the successive stages of a definite progress towards some assigned mode of abstraction from the realm of possibility, involving a progress (in thought) to successive grades of increasing complexity. I will call any such route of progress 'an abstractive hierarchy.' Any abstractive hierarchy, finite or infinite, is based upon some definite group of simple eternal objects. This group will be called the 'base' of the hierarchy. Thus the base of an abstractive hierarchy is a set of objects of zero complexity. The formal definition of an abstractive hierarchy is as follows:*
>
> *An 'abstractive hierarchy based upon* g*' where* g *is a group of simple eternal objects, is a set of eternal objects which satisfy the following conditions,*

> *(i) the members of* g *belong to it, and are the only simple eternal objects in the hierarchy,*
>
> *(ii) the components of any complex eternal object in the hierarchy are also members of the hierarchy, and*
>
> *(iii) any set of eternal objects belonging to the hierarchy, whether all of the same grade or whether differing among themselves as to grade, are jointly among the components or derivative components of at least one eternal object which also belongs to the hierarchy.*
>
> *It is to be noticed that the components of an eternal object are necessarily of a lower grade of complexity than itself. Accordingly any member of such a hierarchy, which is of the first grade of complexity, can have as components only members of the group* g*, and any member of the second-grade can have as components only members of the first grade and members of* g*; and so on for the higher grades.*
>
> *The third condition to be satisfied by an abstractive hierarchy will be called the condition of connexity. Thus an abstractive hierarchy springs from its base; it includes every successive grade from its base either indefinitely onwards, or to its maximum grade; and it is 'connected' by the reappearance (in a higher grade) of any set of its members belonging to lower grades, in a function of a set of components or derivative components of at least one member of the hierarchy.* (167-68)

Here we have the definition of an abstractive hierarchy. To get a sense of this, let us return to mathematics. The base of the abstractive hierarchy of mathematics are the

numbers: (i) all numbers belong to mathematics, and they are the only simple eternal objects in the hierarchy; (ii) Any proposition about numbers (i.e., any complex eternal object concerning the relations among number) can be factored into components that are either propositions about numbers, or numbers themselves; and (iii) any complex proposition about numbers will be the element of some more complex proposition about numbers. Thus mathematics is an abstractive hierarchy defined with "number" as a base. Other abstractive hierarchies, such as those characterizing sensory objects, include simple eternal objects such as sensa in their bases. They, nonetheless, share the same structure as do purely abstract hierarchies like mathematics.

> *An abstractive hierarchy is called 'finite' if it stops at* a *finite grade of complexity. It is called 'infinite' if it includes members belonging respectively to all degrees of complexity.*
>
> *It is to be noted that the base of an abstractive hierarchy may contain any number of members, finite or infinite. Further the infinity of the number of the members of the base has nothing to do with the question as to whether the hierarchy be finite or infinite.* (168)

It can be puzzling to realize that even an abstractive hierarchy which has a base consisting of a finite number of objects can still be infinite. But we can multiply relations among elements, and relations among relations among elements, and so on, with no possible end, even if the number of elements itself is finite.

> *A finite abstractive hierarchy will, by definition, possess a grade of maximum complexity. It is characteristic of this grade that a member of it is a component of no other eternal object belonging to any grade of the hierarchy. Also it is evident that this grade of maximum complexity must possess only one member; for otherwise the condition of connexity would not be satisfied.* (168)

Whitehead earlier in the chapter, defines connexity as follows: "any set of eternal objects belonging to the hierarchy, whether all of the same grade or whether differing among themselves as to grade, are jointly among the components or derivative components of at least one eternal object which also belongs to the hierarchy." In the case of a finite hierarchy, there must be just one eternal object of which all the other eternal objects are components or derivative components.

> *Conversely any complex eternal object defines a finite abstractive hierarchy to be discovered by a process of analysis. This complex eternal object from which we start will be called the 'vertex' of the abstractive hierarchy; it is the sole member of the grade of maximum complexity. A grade which is one lower than that of a given eternal object will be called the 'proximate grade' for that object. We take then those components of the vertex which belong to its proximate grade; and as the second stage of the analyzed them into their components. Among these components there must be some belonging to the proximate grade for the objects thus analysed. Add to them the components of the vertex which also belong to this grade of 'second proximation' from the vertex; and, at the third stage analyze as before. We thus find objects belonging to the grade of third proximation from the*

> *vertex; and we add to them the components belonging to this grade, which have been left over from the preceding stages of the analysis. We proceed in this way through successive stages, till we reach the grade of simple objects. This grade forms the base of the hierarchy.* (168-69)

This paragraph simply says, in a rather complex way, that any given complex eternal object can be considered as the vertex of a finite abstractive hierarcy and can be decomposed, ultimately, into a (finite or inifinite) set of simple eternal objects and a hierarchy of relations among them.

> *It is to be noted that in dealing with hierarchies we are entirely within the realm of possibility. Accordingly the eternal objects are devoid of real togetherness: they remain within their 'isolation.'*
>
> *The logical instrument which Aristotle used for the analysis of actual fact into more abstract elements was that of classification into species and genera. This instrument has its overwhelmingly important application for science in its preparatory stages. But its use in metaphysical description distorts the true vision of the metaphysical situation. The use of the term 'universal' is intimately connected with this Aristotelian analysis: the term has been broadened of late; but still it suggests that classificatory analysis. For this reason I have avoided it.* (169)

These issue have been previously discussed.

> *In any actual occasion* a, *there will be a group g of simple eternal objects which are ingredient in that group in the most concrete mode. This complete ingredience in an occasion, so as to yield the most complete fusion of individual essence with other eternal objects in the formation of the individual emergent occasion, is evidently of its own kind and cannot be defined in terms of anything else. But it has a peculiar characteristic which necessarily attaches to it. This characteristic is that there is an infinite abstractive hierarchy based upon* g *which is such that all its members are equally involved in this complete inclusion of* a. (169)

If we consider a natural entity in terms of a classificatory scheme, we can imagine that it is adequately characterized by its genus and species in that scheme. The entire apparatus of classification sorts entities in terms of their inclusion in a finite abstractive hierarchy. But, in actuality, in each occasion there is some group of simple eternal objects that are included in a very high grade of inclusion, and the relations among those simple eternal objects form an *infinite* abstractive hierarhcy. Whereas a classificatory scheme suggests that an actuality can be fully characterized by a finite set of propositions, in fact it takes an *inifnite* number of propositions to characterize any finite entity. Whitehead is here pointing out that a simple classificatory scheme is entirely inadequate to the true complexity of actual events. With this awareness of the extreme simplification involved in classification, it is easier to see that classification is relevant primarily to the classifier. The characteristics which lump entities into classes are very thin abstractions from the full character of the entities involved. We thus are

permitted to enquire as to the "why" of that particular abstraction, and the answer to that question can only be found in the experience of the entity doing the abstracting.

> *The existence of such an infinite abstractive hierarchy is what is meant by the statement that it is impossible to complete the description of an actual occasion by means of concepts. I will call this infinite abstractive hierarchy which is associated with* a *'the associated hierarchy of* a.*' It is also what is meant by the notion of the connectedness of an actual occasion. This connectedness of an occasion is necessary for its synthetic unity and for its intelligibility. There is a connected hierarchy of concepts applicable to the occasion, including concepts of all degrees of complexity. Also in the actual occasion, the individual essences of the eternal objects involved in these complex concepts achieve an aesthetic synthesis, productive of the occasion as an experience for its own sake. This associated hierarchy is the shape or pattern, or form, of the occasion in so far as the occasion is constituted of what enters into its full realization.* (169-70)

At this point, the elaborate process of definition in which we have been engaged begins to bear fruit. We see that each occasion includes some set of eternal objects as relevant to its aesthetic synthesis. Then, the ultimate character of each actual entity is just the infinite abstractive hierarchy built on that base. This statement points out a significant epistemological implication of Whitehead's metaphysical position. The abstractive hierarchy associated with any actual occasion is infinitely complex. Thus as Whitehead says in the preceeding paragraph, "it is impossible to complete the description of an actual occasion by means of concepts." An actual occasion is *infinitely* complex, but our

minds (as will be discussed shortly) can only deal with finite abstractive hierarchies. Thus all of our knowledge of actuality is only partial. We know reality by means of abstractions from it.

> *Some confusion of thought has been caused by the fact that abstraction from possibility runs in the opposite direction to an abstraction from actuality, so far as degree of abstractness is concerned. For evidently in describing an actual occasion* a, *we are nearer to the total concrete fact when we describe a by predicating of it some member of its associated hierarchy, which is a high grade of complexity. We have then said more about* a. *Thus, with a high grade of complexity we gain in approach to the full concreteness of* a, *and with a low grade we lose in this approach. Accordingly the simple eternal objects represent the extreme of abstraction from an actual occasion; whereas simple eternal objects represent a minimum of abstraction from the realm of possibility. It will, I think, be found that, when a high degree of abstraction is spoken of, abstraction from the realm of possibility is what is usually meant -- in other words an elaborate logical construction.* (170)

In the realm of abstraction, simple eternal objects are the more concrete (in Whitehead's sense of the "concrete" as the full, complete, and divisible but undivided). A simple possibility leaves everything else open to determination. The wider the field of possibility, the greater potential it holds, and the closer it is to the fullness of possibility itself. For example, to describe an entity as "actual" leaves every other character that it

possesses open to further determination. As we add further specificity – "it is red and round," – the range of possibility described is much narrower, and more abstract.

In the realm of actuality this is reversed. The more fully we can describe the abstractive hierarchy of an occasion, the closer we are to its concrete actuality, whereas the simpler our description, the more abstract it is. For example, to say "that is a ball" is a very abstract description. But to say "that is a red ball that is 6" in diameter" is more concrete.

This is not a mere function of language, but a deep truth about the relationship between possibility and actuality. There is a diminishment of possibility with an approach to complexity of relations. The absolute zero of this continuum is actuality, or complete determination. What is thus determined is infinitely complex. To abstract from actuality, we move into the zone of ever simpler and ever less constrained possibilities. To abstract from possibility, we move towards greater complexity, and specificity.

> *So far I have merely been considering an actual occasion on the side of its full concreteness. It is this side of the occasion in virtue of which it is an event in nature. But a natural event, in this sense of the term, is only an abstraction from* a *complete actual occasion. A complete occasion includes that which in cognitive experience takes the form of memory, anticipation, imagination, and thought. These elements in an experient occasion are also modes of inclusion of complex eternal objects in the synthetic prehension, as elements in the emergent value. They differ from the concreteness of full inclusion. In a sense this difference is*

> *inexplicable; for each mode of inclusion is of its own kind, not to be explained in terms of anything else. But there is a common difference which discriminates these modes of inclusion from the full concrete ingression which has been discussed. This* differentia *is* abruptness. *By 'abruptness' I mean that what is remembered, or anticipated, or imagined, or thought, is exhausted by a finite complex object. In each case there is one finite eternal object prehended within the occasion as the vertex of a finite hierarchy. This breaking off from an actual illimitability is what in any occasion marks off that which is termed mental from that which belongs to the physical event to which the mental functioning is referred.* (170-71)

The distinction suggested in this paragraph is an entirely original way of separating actualities from the subjective experience of those actualities. It is further developed below.

> *In general there seems to be some loss of vividness in the apprehension of the eternal objects concerned: for example, Hume speaks of 'faint copies.' But this faintness seems to be a very unsafe ground for differentiation. Often things realized in thought are more vivid than the same things in inattentive physical experience. But the things apprehended as mental are always subject to the condition that we come to a stop when we attempt to explore ever higher grades of complexity in their realized relationships. We always find that we have thought of just this -- whatever it may be -- and of no more. There is a*

> *limitation which breaks off the finite concept from the higher grades of illimitable complexity.* (171)

What Whitehead is suggesting is that when we prehend, are causally affected by, or notice an actuality within our experience, that which we prehend is always characterized, for us, by a finite abstractive hierarchy. Every event is a mystery. We can always learn more about it. But our ideas about events are just what they are. They can be fully characterized by a finite abstractive hierarchy.

> *Thus an actual occasion is a prehension of one infinite hierarchy (its associated hierarchy) together with various of finite hierarchies. This synthesis into the occasion of the infinite hierarchy is according to its specific mode of realization, and that of the finite hierarchies is according to various other specific modes of realization. There is one metaphysical principle which is essential for the rational coherence of this account of the general character of an experient occasion. I call this principle, 'The Translucency of Realization.' By this I mean that any eternal object is just itself in whatever mode of realization it is involved. There can be no distortion of the individual essence without thereby producing a different eternal object. In the essence of each eternal object their stands an indeterminateness which expresses its indifferent patience for any mode of ingression into any actual occasion. Thus in cognitive experience, there can be the cognition of the same eternal object as in the same occasion having ingression with implication in more than one grade of realization. Thus the translucency of realization, and the possible multiplicity of modes of ingression into the same*

occasion, together form the foundation for the correspondence theory of truth.[28]

(171-72)

In terms of the understanding that we are developing here:

- The actual world is a field of differentiations the richness, depth and complexity of which transcends the grasp of finite thought.
- Thought can abstract from that fullness a set of simple eternal objects as the base, and a finite abstractive hierarchy as the character of those entities by which it is affected (which it prehends).
- Thought forms its abstractive hierarchies by abstracting, from the entity which it prehends, some eternal objects that are, in fact, in that entity itself. It is not a free invention of mind, but a refinement of prehensive process. But various prehensive processes differ in fidelity and adequacy. By virtue of this principle, thoughts are actually connected to the entities to which they refer, and can be more or less true.
- It is always an error of misplaced concreteness to confuse a finite hierarchy with that of which it is, in part, the character.

In this account of an actual occasion in terms of its connection, with the realm of eternal objects, we have gone back to the train of thought in our second chapter, where the nature of mathematics was discussed. The idea, ascribed to Pythagoras, has been amplified, and put forward as the first chapter in metaphysics. The next chapter is concerned with the puzzling fact that there is an actual course of events which is in itself a limited fact, in that metaphysically

> *speaking it might have been otherwise. But other metaphysical investigations are omitted; for example, epistemology, and the classification of some elements in the unfathomable wealth of the field of possibility. This last topic brings metaphysics in sight of the special topics of the various sciences.* (172)

Epistemology has not been fully explored here, as it will be in *Process and Reality.* Also, this essay has been metaphysical, rather than cosmological, so that specific groups of possibilities – such as, for example, the possibilities for the types of events that are studied in physics, law, or psychology – have not been singled out and explored.

END NOTES

[1] Alfred North Whitehead, *Science and the Modern World*, New York: The Free Press, 1967.
[2] Alfred North Whitehead, *Process and Reality*, New York: The Free Press, 1978.
[3] The work done on this commentary was supported by a grant from the Esalen Center for Theory and Research.
[4] Alfred North Whitehead, *An Enquiry Concerning the Principles of Natural Knowledge*, p. 59.
[5] The question concerning the 'how' of the transcendent existence of the eternal objects is not addressed in this essay. In *PR*, Whitehead makes the existence of the eternal objects a metaphysical presupposition, and arranges for them to be ordered as apt for realization in the actual world through a primordial concrescence issuing as the Primordial Nature of God.
[6] When I assert that each actual occasion "thinks," what I mean is that each actual occasion considers possibililities and decides among them.
[7] Of course colors, sounds, and other simple eternal objects can be analyzed scientifically in terms of various sorts of "vibration." But the fact that we can associate a particular shade of red, under suitable circumstances, with a particular frequency of electromagnetic radiation tells us something important about the nexus of causal relations disclosed in perception, but it tells us nothing about the redness of red.
[8] Later in this essay, Whitehead refers to this property of eternal objects as the 'Translucency of Realization.'
[9] This is called, in *PR*, the "Principle of Relativity."
[10] Paul Edwards (editor in Chief) *The Encyclopedia of Philosophy*, New York:Macmillan Publishing, Inc., 1967 - article on "Relations, Internal and External" in Volume 7, p. 125.
[11] My attention was brought to this issue by Dr. Michael Epperson, who makes extensive use of this principle in his very valuable book: Michael Epperson, *Quantum Mechanics and the Philosophy of Alfred North Whitehead*, NewYork:Fordham University Press, 2004.
[12] The long and difficult process through which the modern idea of spacetime emerged is documented in Ivor LeClerk, *The Nature of Physical Existence*, New York: Humanities Press, Inc., 1972.
[13] This essay, "Uniformity and Contingency, is found in Alfred North Whitehead, *The Interpretation Of Science: Selected Essays*, New York, Bobbs-Merrill, 1961, pp. 108-124. I find it to be of interest that in this essay, on page 112, Whitehead discusses his experiences of a lucid dream.
[14] Ibid., p. 113.
[15] We may say that the dream took place between my going to sleep and my awakening. This locates the event of the dream within the dominant continuum. But the time I experience in the dream is, nonetheless, not part of that continuum.
[16] This modification is important because it can help us to understand the actuality of the places that we visit in lucid dreams, out-of-body experiences, and after death. I have explored this in Eric Weiss, *The Doctrine of the Subtle Worlds*, UMI Dissertation services, 2003.
[17] The following remarks draw heavily on Morris Kline, *Mathematics, the Loss of Certainty*, New York, Oxford University Press, 1980.

[18] Ibid., 114-115.
[19] Ibid., p. 78.
[20] Saccheri himself derived a theorem so strange to him that he assumed he *had* derived a contradiction, but subsequent mathematicians realized that the strangeness of the theorem he derived was not, in fact, a contradiction., Ibid., p. 80.
[21] Ibid., p. 81.
[22] Whitehead's argument concerns the meaning of simultaneity and the meaning of measurement. He felt that Einstein's notion of simultaneity was defective in that it *assumed* the constancy of the speed of light and also because it assumed a specious present in which simultaneity can be noted but still treats actuality as a succession of instants. Also, Whitehead realized that measurement is meaningful only if spacetime is of uniform curvature. By envisioning spacetime as having variable curvature, Einstein had made the very measurements on which his theory depends impossible. For a further elaboration of these ideas, see Alfred North Whitehead, *The Principle of Relativity with Applications to Physical Science* in F.S.C. Northrop (ed.), *Alfred North Whitehead, an Anthology*, Macmillan Company:New York, 1961, pp.295-358.
[23] Weiss, Op. cit.
[24] Bertrand Russell, *Introduction to Mathematical Philosophy*, London: George Allen and Unwin Ltd., 1953 (originally published in 1919).
[25] I'm assuming that "successor of" is of grade 1, and that those axioms which refer to it are of grade 2. I will readily concede that there might be some debate as to whether or not the induction axiom can be graded in the precise sense to which Whitehead is referring. It moves, in some way, to the edge of the infinite, and therefore creates questions that the other axioms do not.
[26] The idea of a "personally ordered society of occasions" comes from *Process and Reality*. It is a society of occasions that involves only one occasion in a given moment. It moves through successive concrescences like beads on a string. All personalities, in Whitehead's sense of the term, are personally ordered societies.
[27] Henri Bortoft, *The Wholeness of Nature : Goethe's Way Toward a Science of Conscious Participation in Nature*, New York:Lindisfarne Books, 1996.
[28] Whitehead says that an actual occasion is, among other things, the prehension of one infinite hierarchy. This idea seems somewhat problematic in light of the fact that thought involves operations with finite abstractive hierarchies, and that thought is involved in the decisions leading to the aesthetic synthesis. It would seem to me to be more reasonable to say that the final, and most complex proposition entertained by a concrescing occasion establishes the base of an infinite abstractive hierarchy that it implies. The ultimate character of an occasion is richer than it can know. No artist can detect all of the meaning in her creation.