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THE EXPERIENCE OF SPACE: *Chaos Theory and the Evolution of Consciousness*

Term paper for *Epic of Evolution* and for *Chaos, Complexity and Consciousness*

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I. INTRODUCTION

- A. This paper is the beginning of a meditation on the notion of phase space.
- B. I have the intuition that the notion of phase space is much more than a computational convenience I believe, rather, that it is a pivotal notion in the development of Western thought. If I am correct, the contemplation of the notion of phase space can bring clarity about both the explanatory power and the blinding limits of the scientific enterprise in general, and about the science of complexity in particular.
- C. This paper attempts to begin the meditation on phase space by contemplating the evolution of consciousness which makes the notion of phase space possible, and by defining phase space within that context.

II. PHASE SPACE – PRELIMINARY CONSIDERATION

- A. The task of defining phase space properly will only become possible as we proceed further into this meditation. However we need some kind of tentative definition in order to get us started, and in order to provide some kind of tentative definition, we need a context in which to operate.
- B. Let me begin by remarking on a move that Albert Einstein and Leopold Infeld make at the very beginning of their book *The*

- Evolution of Physics*.¹ They, too, are looking for a place to begin, and they do so by bringing attention to the problem of *motion*. They point out that motion in physical space is, in fact, quite intricate. They note the inadequacies of the Aristotelian analysis, and they sing praises to Galileo for the way in which he brought fresh intuitions and a new mathematical rigor to this particular investigation. For Einstein and Infeld, the evolution of modern physics begins with the application of mathematical tools to the problem of producing exact descriptions of the movements of objects in physical space.
- C. Having said that, let's come back to phase space. Phase space is a central notion, perhaps *the* central notion, in the science of *dynamics*. And dynamics is a set of mathematical tools for describing and analyzing various kinds of movements in various kinds of space. So without yet defining phase space, we can at least say that phase space has meaning in the context of dynamics, and that dynamics has meaning in the context of physics.
- D. But let's slow down a little bit, and not get carried away. Isn't it a bit odd that Einstein and Infeld should begin a book on the evolution of physics with the problem of *motion*? Why is motion in physical space so interesting? Why should we care about measuring it so carefully? And why should we imagine that applying mathematical analysis to the movements of objects in space should somehow lead to great revelations about the nature of reality? None of this is intuitively evident.
- E. This is one of the points at which the 'two cultures' tend to diverge. It is one of the many points at which humanities students tend to 'bounce off' of scientific reasoning. Before we can appreciate the power of descriptions in terms of phase space, we have to experience the allure of dynamics. And before we can appreciate dynamics we have to surmount two preliminary hurdles:
1. First, we have to get interested in the problem of motion in physical space
 2. Secondly, we have to grasp the way in which mathematical tools can help us to clarify that problem.
- F. These hurdles exist for humanities students – even here in the late twentieth century. And they existed, obviously, for human beings prior to the 1600's. I think that the difficulty that humanities students have with dynamics is closely related to the situation of pre-modern humanity as a whole. As a way of further pursuing our

meditation on phase space, then, I would like to consider this question: Why *wasn't* the mathematical analysis of motion in physical space interesting to educated people before the time of Galileo?

- G. The simple answer to this question, one that might tempt a thoroughly modern scientist, is that the problem was just too difficult, and that the tools hadn't yet been developed.² But this answer should fail to satisfy us because it involves circular reasoning. The mathematical tools that are required for describing motion *evolved, at least in part, in the context of the attempt to describe motion.*³ It's too simple to say that the problem couldn't be solved because the tools didn't exist. The tools didn't exist because people didn't find the problem interesting enough to bother to develop the tools that they would need to solve it.
- H. This brings us back to our question. What were people up to before the modern age? What were they doing with their intellects that kept them from bothering themselves overmuch with dynamics? And what changed to make the problem of motion emerge as a focus of such intense interest?

III. THE PREHISTORY OF PHASE SPACE – MYTHIC SPACE

- A. Because space, as a concept and as an experience, is so pervasive, we tend to take it for granted. We assume that our experience and our concepts tell us something about '*the*' nature of space. A strong case can be made, however, for the idea that space is an historical and cultural reality.⁴
- B. When Einstein and Infeld speak about the problem of motion, they take it for granted that motion is *the displacement of objects in physical space*. Within the context of a modern consciousness, that definition is obvious. Prior to modern times, however, (and deep in the psyches of us pre-scientific humanities students) that definition is far from obvious. I think that for pre-moderns, both motion and space actually had different meanings from the meanings that they now have.
- C. I would like to see if we can imagine ourselves into a pre-modern sense of space. I am not trying to do a specific piece of historical analysis, but rather to evoke a coherent sense of just how a pre-modern sense of space *could* have worked. My objective is not so much to speculate on how things were, but rather to use a fantasy of the past to provide a reference point from which we can get a better perspective on the present.

D. Imagining a pre-modern space⁵

1. Let us imagine ourselves as a couple of our ancestors, sitting on a small rise, beneath an old tree, looking down on our village. We are somewhere in Europe, sometime before 1400. Let us imagine that we have never been more than five miles from here. We have never seen a map, never read a book. There's not much to do this afternoon, and we're in a philosophical mood, wondering about the greater world that we've never seen.
 2. Local space, the space in which we live day to day is, in some ways, not too different from the space that moderns will someday inhabit. We know how to throw a ball and catch it, we know how to walk from here to there. We have a reasonable sense of distance and of basic, pragmatic geometry.
 3. But as we let our imaginations move out to greater and greater distances, something happens that is rather strange to modern sensibilities. As we extend our imaginations into the further reaches of space, that space becomes more and more dreamlike. All of the information that we get about far away places comes to us as story. These stories are the only data that we can possibly get about those other places. And those stories tell us of realities that are stranger and stranger as the distance that they tell of becomes greater and greater.⁶ Even if I make a long journey myself, the time and the difficulties involved guarantee that the journey will transform me and all of my perceptions. If I come back at all, it will be as a changed man, with strange stories to tell.
 4. It seems quite natural to us, as we sit here under the warm Sun in this little island of normality that we inhabit, that the further away we go the stranger things will be. At a medium distance, there will be people speaking strange languages and living according to strange customs. At greater distances, the creatures will become more and more magical. At very great distances, we will find the home of gods and demons.
 5. This world of myth and magic is, for us, the *real* world.
- E. It is important that we Moderns do not dismiss too quickly this sense of space. First of all, let us realize just how *real* that sense of space would be to those ancestors of ours. In the modern world, we live in a *real* space of hard vacuum, immensely vast distances, stellar infernos, and so forth. But we lay people, we know about those realities from stories, from pictures, from experts. Our ancestors,

- sitting on that hill overlooking their village, they also knew about strange and distant realities from stories, from pictures, from experts. They knew then about unicorns, ghosts, spirits, dragons, heavens, hells and gods by relying on the same kind of authority that we moderns rely on to know about quarks and black holes. In those days, the people who believed in what we consider to be mythic realities were the sober, practical authorities. That other space, mythic space, was *real*.
- F. Let us look a little more closely at the nature of that space. One way for us Moderns to develop an appreciation for this space would be to look at something I want to call dream space. Let me define what I mean by dream space.
- G. A definition of dream space
1. Let us imagine that we could collect and catalog all possible dreams⁷. These dreams would differ from each other along a large number of parameters. Some dreams would be very concrete, some very abstract. Some dreams would be very close to waking life, some would be very bizarre. Some dreams would be terrifying, some would be sublime, and so forth. We could imagine some kind of catalog system that would let us assign a value to each dream along each of this infinite set of parameters.
 2. Now, suppose that we want to orient ourselves in this dream space. It will be nice to have some kind of zero point or reference point around which we can construct our order. Let us, then, stipulate that the center of this space is roughly defined by our experience of waking life. In this way of arranging dreams, those dreams that are most concrete, most literal, most like our waking life in all respects are those that are closest to the center. Those dreams that are more intense, more abstract, more bizarre, etc. than our waking life are arranged at greater and greater distances from the center.
 3. This space of all possible dreams, oriented around a general center point resembling waking, physical life, is what I want to call dream space⁸.
- H. With this definition in hand, we are now in a position to say something very interesting about the mythic space we are imagining to have been inhabited by our ancestors. That space is a conflation of local (more or less Newtonian) space with dream space. In other words, in mythic space, any sufficiently large movement in physical space is simultaneously a displacement in dream space (if I walk far

- enough, I will get to Mt. Olympus). And a sufficient displacement in dream space will, from any point in that space, bring one arbitrarily close to the local, physical world. The physical world, while it is uniquely privileged as the most continuous and literal of spaces, is nonetheless meaningfully situated within dream space. And, when we assume that movement in physical space brings us into more and more dreamlike realities, physical movement is understood as a kind of special (and not very interesting) case of movement in dream space dimensions.
- I. We can see then, that in a reality in which any reasonably large movement in space is assumed to involve a fundamental shift in the nature of the reality that we are experiencing, the precise measurement of physical movements is not an issue which attracts a great deal of attention.
 - J. In mythic space, physical dimensionality has a very restricted meaning. Those of you who are already familiar with phase space will recognize that what I am calling dream space has a certain distant kinship with phase space⁹. But the various parameters which define dream space cannot be measured mathematically. Now, if we cannot assign numbers to the various parameters that order dream space, how can we make sense of it? We make sense of dream space by telling stories. The active agents in stories are subjects, conscious beings with active intentionality. There is very little in dream space that is measurable. But everything in dream space is meaningful. And meaning, a fundamentally subjective category, is both apprehended and given by subjects.
 - K. Now, let's go back to that hill overlooking the village. We are sitting there on this sunny afternoon, and to our senses it doesn't look too different than it would look today. But as we are sitting there many hundreds of years ago, we *know* that this local island of physical normality is situated in a vast ocean of meaningfully ordered dream space, and that dream space is given its meaningful structure by the activities of conscious beings. This knowledge will have a significant effect on the way that we interpret our perceptions in local space.
 - L. For us moderns, the causal forces that operate in physical space are automatic structuring factors that move in geometrical forms. In mythic space, the causal factors are animate presences (Devas, Spirits, Angels, Archangels, Thrones, Souls . . .). With the expectation that the forces structuring events are animate, human or superhuman I will notice entirely different categories of events than I will notice if

- I assume that the causal forces are dead and automatic. In modern space, we are hyperconscious of *things* and their measurable interactions. In mythic space, we are primarily conscious of situations and transactions that we can capture with stories of purpose and intent.
- M. In mythic space, those parts of our being which resonate with the larger, more ultimate realities – our dreams, our visions, our consciousness, will, when we engage in speculative thought, be more interesting to us than the pragmatic details of everyday life. And even when we consider everyday life, the problem of assigning exact numerical definitions to the movements of physical objects will have a very low priority.
- N. In mythic space, synchronicities will capture our attention much more vividly than will regular causal interactions.¹⁰ In that space, meaningful coincidence will point beyond the everyday and will provide a situation in the context of which the operation of the primary causal factors which we hold to be governing the real can be examined. As we will see, the role of synchronicities will, in the modern space, be replaced by ‘experiments’.
- O. What I have been trying to evoke in this imaginal prehistory of phase space is a situation in which intelligent, thoughtful people will find it odd and peculiar to spend a great deal of intellectual energy trying to measure and analyze the movements of objects in physical space. I might be so bold as to imagine that the space that I am projecting back into the pre-modern past is not that different in some respects from the space in which many of us humanities students often dwell.
- P. So what happens to open up the modern world, the modern space, the space in which the problem of motion suddenly acquires its vast import?

IV. MORE ON THE PREHISTORY OF PHASE SPACE – THE EMERGENCE OF PERSPECTIVAL SPACE

- A. The move from mythic space to modern space is what Gebser calls a ‘mutation’ of consciousness.¹¹ It is a deep and extensive change which affects every area of life. But we can get a very clear sense of what this mutation of consciousness is about by contemplating the invention of linear perspective drawing in the fifteenth century.
- B. All of us educated moderns are familiar with perspective drawing. It is so familiar to us, it seems so natural to us, that we assume that space *really* is the way it looks in perspective. But there are *no*

examples of drawings or paintings done in perspective in *any* culture whatsoever prior to fifteenth century Italy.¹² If space *really* were the way we moderns imagine it to be, then it seems that other peoples in other times would have found means to depict that reality. It seems, however, that this sense of space out of which we operate is something which emerged into consciousness at a particular place and at a particular time in history. And this space which is depicted through linear perspective is precisely the space which enables and encourages the particular attention to the problem of motion that lies at the heart of dynamics.

- C. Perhaps you can remember the wonder you felt as a child when you first learned how to ‘see’ the third dimension in a plane. It was as if there were a wondrous, magical space somehow sandwiched into the plane of the drawing. Perhaps you turned over the page you were looking at in an attempt to discover the ‘inside’ of the depths that you were perceiving. Even today, when we are totally saturated with perspective, we can revive a sense of wonder about it with ‘stereograms’ or holographic images. Let us, as a part of our meditation, take the time to compare and contrast that strange and abstract space ‘inside’ the perspective drawing with the rich and complex space in which we actually live.
- D. Exploring perspectival space
1. The space in which we actually live completely surrounds us. We are inside of it. We are vulnerable in this space. We can be affected by whatever is here. On the other hand, we are always and forever outside of the space of perspective. To make a perspective drawing, we imagine ourselves looking at the scene through a window. We are outside of the scene, looking in. We are safe, removed, at a distance. By definition, the observer of a perspective drawing is never *in* the scene. The invention of linear perspective drawing is inseparable from the invention of the objective observer – detached, unaffected, a spectator.
 2. In lived space, objects are alive with significance. Objects in lived space enter our awareness as we live through them. They are inherently charged with situational significance. They are mine or yours. They are familiar and intimate, or charged with alien intent. In perspective space, objects begin to reveal themselves in a kind of naked facticity. Objects in perspective become, in a later development of the same way of seeing, objects in museums, pulled out of context, encased in glass,

cataloged, defined, mastered.

3. The space in which we live is relatively indifferent to measurable size and measurable distance. Think about what it's like when you are first arriving in a new city. At first the whole place is a confusing blur of sights and sounds, but gradually certain reference points emerge. This is the café where I met my friend the day after I arrived. This is the park where I sat and cried because I missed my old home. This is the street where I got lost trying to get to my first job interview. If you were asked to depict the city shortly after you arrived, you could only construct a kind of montage of charged images. And this is, on a deep level, how we perceive our world. It is how our perception of the world is arranged. It is a montage of charged images. Only later, only with training and discipline, can we manage to pull out of the lived world, to find an external vantage point, and draw a map. *In this lived world, size and distance are not mathematical measures, they are signifiers of meaning.* If I had moved to this city to be with my friend, he would be the largest object in any picture I would draw. But in the world inside the perspectival drawing, size and position are no longer primarily expressions of meaning, they are rather measures of spatial distance. In lived space, outside of the drawing, measurable size and measurable distance are often secondary and unimportant. In perspectival space they are always of decisive importance.
4. Lived space is closer to mythic space than it is to perspectival space in that every location in lived space is permeated by a different field of subjective meaning. Movement in lived space is much less a movement from place to place and much more a movement from situation to situation. But inside the perspectival space, one place differs from another in function of its distance from the observer and from other objects in the space. Within the perspectival space, measurable physical distance decisively differentiates itself from the meaning-charged distance of dream space.
5. In lived space, measurement of size and distance apart from their meaning is often quite difficult. The neighboring village is a half-a-day's walk, and to get there I have to cross a rickety bridge and I have to pass through a dark and scary woods. The fact that the neighboring village is 6.3 miles away 'as the crow

flies' is information that is irrelevant and inaccessible to me. But in the space inside the perspectival drawing, size and position are always and intrinsically measurable. To create a perspective drawing, I first produce a structure of lines that *mathematically* defines the space. Everything in that space is, henceforth, related to that framework

6. The mathematical framework of perspectival space extends off to infinity. No matter how large that space becomes, it suffers no qualitative deformation. No matter how far we go in perspectival space, we never find the dwelling place of the gods
- E. There is a great deal more that can be said about perspectival space. Both Romanyshyn and Gebser explore this theme in great depth.¹³ But we have, I hope, said enough for our current purpose. In looking at linear perspective, in contemplating the space that opens up, as a new development on planet Earth, *inside* the drawing, we have an image that allows us to describe the birth of the modern consciousness.
- F. The universe that people depict in the artistic creations is the universe that they inhabit. And, of course, people come to inhabit the universe that they see depicted. The bizarre and unreal space that opens up inside the plane of the perspectival drawing becomes gradually the space in which all of us live and move and have our being.
1. The strange Cartesian cogito at the core of the modern ego is the spectator of the perspectival painting, somehow and paradoxically finding himself inside of the scene which he simultaneously transcends (the male pronoun is, perhaps, particularly excusable in this context).
 2. The space in which placement of objects is relative to a set of artificial lines becomes Newtonian absolute space. This space reaches out and subsumes time, which itself becomes a grid against which measurements can be made.
 3. The *still life*, in which a scene is reduced to measurable distances against a regular grid, morphs into the *experiment* in which a system is reduced to a set of measurable properties.
- G. We saw that in pre-modern mythic space (at least as we imagined that space to have functioned), dream space and local space were conflated. With the rise of modern consciousness, these spaces become clearly differentiated.
1. When we understand local space as 'the space inside of the

painting,' then there is no place where local space merges into imaginal space. Like perspectival space, it just goes on and on forever, never changing its laws.

2. Because local space no longer borders on dream space, it begins to seem less and less likely that the forces structuring local space are living and intentional. In trying understand local objects, we become less interested in their *meanings* and more interested in their size, in their positions, and, by extension, in their other measurable properties.
 3. In this context, dream space begins to seem less and less real. It retreats into a shadowy existence somewhere behind the observing subject and there it becomes what we later call the Unconscious.
- H. Thus perspectival space, as it erupts into Earth reality through human culture, becomes the all enveloping world of cosmic space in which scientific consciousness functions. *It is in perspectival space, the space inside of the painting, a space which has escaped the borders of the frame and swallowed the artist whole, that the problem of motion in its modern form first becomes deeply interesting.*
- I. We are considering here the history of phase space. We have established that phase space, whatever it is, is part of dynamics, or the mathematical study of motion. We now see that dynamics could only be conceived by a modern consciousness situated in a perspectival space.
- J. The problem of motion in physical space had, of course, always been somewhat interesting. People in the pre-modern world were involved in trade, in war, in construction, and all of these activities involved problems of motion. But prior to modern times, the problem of motion held no deep theoretical interest. In pre-modern times, deep thinkers thought about the divine. Only the riff-raff bothered themselves with the trivialities of measuring motion. But when human beings, like Alice in her looking glass world, woke up within the perspectival painting, surrounded by naked objects emerging against a metrical background, all of a sudden measurement took on a new significance. Now deep thinkers suddenly had a common interest with craftsmen, with traders, with soldiers. It is in perspectival space that the military-industrial complex is born.
- K. We are almost ready, now, to speak about dynamics. There is one other piece that we need to put in place. It is in perspectival space

that objects begin to emerge against a metrical background. In is in perspectival space that measurement of motion becomes both possible and important. But this space not only places objects in a mathematical context, it also transforms the significance of numbers themselves.

- L. As we know, mathematical ideas are found at the very roots of Western civilization. But numbers, like objects, had, up until the Renaissance, been deeply embedded in mythical space. The intuition that numbers are fundamental to the constitution of the real is a very old one, but numbers can be seen as both quantitative and qualitative. In a sense, each number is an archetypal field that is saturated with subjectively interesting meanings.¹⁴ In perspectival space, however, number loses most of its archetypal resonances and emerges above all as quantitative. That aspect of number which has its native home in dream space splits off and becomes numerology. Mathematics, now purged of numerology, studies the quantitative significance that is left. Numbers in perspectival space do not provide contexts of meaning, they provide measurements pure and simple.
- M. We can now begin to see something of what it takes to set the stage for the emergence of dynamics as a science.
1. We have to invent and enter into perspectival space
 2. We have to identify local space with perspectival space, separating it from dream space and extending it off into infinite distances.
 3. We have to discover objects as naked things moving in this metric space, and we have to separate the position and the size (as well as the other measurable attributes of the object) from the subjective significance of the object in our life worlds.
 4. We have to discover number as naked quantity, separating it from its numinous penumbra of archetypal significance.
- N. I think that each and every one of us has to go through these steps if we are going to have a true appreciation for the power and the limits of dynamical science. I suspect that it is our failure to appreciate the immensity of these steps (which many of us take in childhood) that makes the gap between the two cultures so forbidding.

V. THE HISTORY OF PHASE SPACE

- A. Let us see if we can trace how the transition into perspectival space leads quite naturally to curiosity about motion and to the development of the methods of dynamics.

- B. It is no coincidence that within a few hundred years of the advent of perspectival space, there was a general revival of atomism. Atomism is a way of describing physical reality as the movement of particles in space, and this is the description of reality that best suits perspectival vision. In the mythic world, in the life world, the thingness of objects tends to be subordinated to the role of those objects in functional networks and to resonances of meaning among subjects and objects, which are encoded in stories. The thing-like characteristics of objects are downplayed in favor of their functional and symbolic significances. But in perspectival space, it is the thing-like characteristics of the object that move to the fore. In this new space, objects emerge as self-contained and discontinuous, in a word as particulate. If space is *really* perspectival, then the objects in that space are *really* separate and self-existing things.
- C. If space is perspectival and objects are things, *then all processes – all birth, all death, all becoming, all events – are essentially movements of things in space.*
- D. Now, I hope, it becomes clear why it is that the problem of movement, for so long a relatively minor outpost of intellectual life, now becomes a major preoccupation. In this new perspectival space, it suddenly seems self-evident that all of reality is a movement of things in space. If all of reality is a movement of things in space, then there is no problem more important than that of understanding that movement. The problem of comprehending the subtleties of movement in space becomes the foundational problem of physics, and physics becomes *the* paradigmatic pattern that all of the other sciences attempt to emulate.
- E. This perspectival space not only sets the problem, it also establishes the language in which the problem will be discussed. As we have already begun to see, the relationship between numeration and perspectival space is a very intimate one.
1. In our imaginal reconstruction of the passage into the modern sense of space, we saw number coming into its modern significance as a means of establishing the structure of perspectival space. When we think of perspectival space, we tend to think of the lines which set up the space by establishing the horizon, the vanishing point, and so on. It is as if the perspectival space is drawn on a systematically distorted grid. As we become accustomed to thinking of the grid in the perspectival space, it becomes quite natural to project that grid

outward into what we now begin to see (precisely by imposing the perspectival grid on it) as “three dimensional space.”

2. In perspectival space, displacement in space ceases to be thought of in terms of a process of transformation, and begins to be imagined as a measurable movement of an unchanging object in the context of a three dimensional grid established by a fixed unit of measure. We begin to think of space as an extension that can intrinsically be broken up into contiguous cubes, and of those cubes as intrinsically countable. The *unit* takes on significance as the unit of measure. We begin to think of a line as a collection of units, and we begin to think of numbers as divisions of a line.
- F. Given this intimate connection, it is quite natural that we would want to understand the movements of the particles in the space in terms of numerical representations. It probably never occurred to Aristotle to explore the nature of movement by making numerical measurements. But Aristotle didn't think of space as a three-dimensional grid. Once space is understood as grid, it suddenly seems as if there is no interesting way *other* than mathematics to describe the properties of movement.
- G. In the context of perspectival space, objects are seen as things, all reality is seen as movement of things in space, and relationships among numbers becomes the obvious language in terms of which to explore that movement. In perspectival space, mathematics becomes the language of reality.
- H. Our meditation has, at last, brought us to dynamics. Dynamics grows out of the attempt to develop the mathematical tools for describing the movements of particles in space. Dynamics is a very pure expression of perspectival consciousness.
- I. I want to pause at this point and to look over the full extent of the territory that we have crossed.
1. We have moved from a universe that is:
 - a) Situated in dream space
 - b) That is structured by the intentions of animate powers,
 - c) And that is best understood through stories
 2. To a universe that is
 - a) Situated in an endless uniform grid,
 - b) That is made up of things moving about in patterns
 - c) And in which those patterns are best described in a language that consists of relationships among quantities.

- J. The shock of the transition, if we allow ourselves to meditate on it, is brutal. Nonetheless, here we are. We have come a very long way from our shaded spot under the tree overlooking our village. And we have a ways yet to go before we reach phase space.
- K. It might help the two cultures in understanding each other to realize these two complementary truths:
1. From the perspective of a consciousness that is operating anywhere near the mythic, the whole enterprise of dynamics is incomprehensible. In a mythic universe, the problem of using relationships among numbers as a language for describing the intricacies of movements among things is not only impossibly difficult, it is also relatively uninteresting.
 2. From within the perspectival consciousness, the importance of dynamics is too obvious to need explaining.
- L. Now, the inherent connection between perspectival space and number has already dictated that mathematics will be the language with which we describe the movement of things in space. That same intrinsic connection now manifests itself in a new way. It establishes space as a natural language for describing mathematical relationships.
- M. The realization that figures in space can be interpreted as representations of relationships among numbers is the essence of Descartes' analytical geometry.
1. He begins with the equivalency that we have seen between units of space and increments of quantity. The line becomes the number line. A line is taken to be an abstract representation of all possible quantities.
 2. Now Descartes takes the step of allowing the line to represent not all possible quantities in the abstract, but rather he allows the line to represent the values that can be taken *by a specific variable*.
 3. He can then erects a second line, perpendicular to the first and allows it to represent all of the values that can be assigned to a second variable.
 4. The plane that emerges through the intersection of these two lines is a new kind of space. It is a space of quantitative relationships. Every possible figure in that plane represents some mathematically expressible relationship among two quantities.
 5. This relationship among variables and axes can be generalized into an arbitrary number of dimensions, so that all relationships

among sets of quantities can be modeled in this space.

- N. We have mapped the movement of particles in space onto systematic changes among sets of quantities. We now map systematic changes in sets of quantities onto spatial figures. This use of spatial figures as a means of representing quantitative relationships has been a key tool in dynamics from its inception.
- O. We see space so thoroughly in terms of its perspectival character that we sometimes lose the important distinction between physical space and the space of analytic geometry (which, I suppose, we might call 'number space'). There is one place where the two spaces almost overlap. If we represent physical space by a three dimensional grid, then the imaginary track left by a particle passing through that space (the 'trajectory' of the particle) will be exactly homologous with a specific line in the three dimensional grid that represents that space. In the case of the movement of a cannonball through space, for example, the line that it traces is very close to the line that we get when we trace a quadratic relationship among two variables. That line is a parabola. Most strikingly, the trajectory of the orbits of the planets turns out to be an ellipse.
- P. There is a certain integral solidity in this climactic expression of perspectival space. Space emerges as a three dimensional grid. Reality is imagined as particles moving through space. The movements of the particles are represented as sets of numbers. The relationships among those numbers are represented as lines drawn in space. Objects are found to move through space leaving imaginary tracks that are homologous to the lines we have drawn. There is a beautiful sense of closure. In those early days of dynamics it was felt that reality had, at last, been understood.
- Q. We are now beginning to close in on phase space, but there is one more major conceptual transition that we have to negotiate.
1. We started out with mythic space, which we came to understand as a conflation of dream space and local space.
 2. We saw that in mythic space, what creates orientation is meaning, that meaning is traced back to the activities of animate and intentional beings, and that it is expressed and understood through stories.
 3. We saw that the eruption of perspectival space thrust dream space into the background (where it become the Unconscious), transformed local space into a metric grid, and extended local space out to infinity.

4. In perspectival space, meaning becomes secondary and ‘merely subjective’. Objects reveal their thing-like characteristics, and reality begins to look like the movements of naked things in metrical space.
 5. The best way to understand reality in this new space is to understand movement; the best way to represent movement is in terms of numbers; and mathematics (understood, now, as a language for expressing relationships among numbers) replaces story as the best way of formulating, for our understanding, the nature of the real.
 6. Finally, the hermeneutic circle in perspectival space becomes closed when Descartes realizes that not only can movement best be represented as changing sets of quantities, but changing sets of quantities can systematically be represented as figures in space.
 7. Whereas in mythic space, there was a conflation of dream space and local space, here we see a kind of overlap between physical space (perspectively understood) and ‘number space’. We find that we can represent movement in space as mathematical expression, and represent mathematical expression as figure in space.
 8. The crucial move that we have to make if we are going to understand phase space is, now, to clearly separate these two spaces.
- R. Unfortunately for classical physics, it quickly became apparent to physicists that they could not account for *all* process exclusively in terms of the movement of particles in space. Phenomena like heat, electricity, magnetism, chemical interactions and so forth just would not yield to an analysis in terms of point particles serenely sailing through space. But by now particles had revealed themselves as possessing measurable properties, and the relationships among those properties had revealed themselves being comprehensible in terms of mappings onto perspectival space. With analytic geometry, perspectival space, which has already revealed itself as an infinite grid against which movements can be measured, now reveals itself as number space, a space in which all possible relationships among sets of numbers can be represented.
- S. The idea that all of reality is a movement of particles in perspectival space becomes abstracted. Reality now begins to be understood as systematically interrelated changes in the values of parameters that

can be represented in perspectival number space.

- T. Number space, an abstract extension of perspectival space into an arbitrary number of dimensions becomes, for modern space, what dream space had been for mythic space. In a reality in which all real relationships can be represented as relationships among quantities, a space in which all quantitative relationships can be represented is the ultimate space of meaning.
- U. Phase space is a particular configuration of number space. We are now, at last, in a position to define phase space.

VI. THE DEFINITION OF PHASE SPACE¹⁵

- A. Phase space is a way in which we can use number space to map all of the possible *behaviors* of a *system*.
- B. In this context:
 1. A system is understood to be an object which we can describe in terms of measurable parameters
 2. Behavior is understood to be variation in and among those measurable parameters.
- C. Now objects are, for us, in our lived worlds, much more than collections of measurable parameters. Nonetheless, we are now operating in perspectival space, and in that space we have become accustomed to thinking of objects as things, and we have come to think that the *real* properties of things are those that are measurable. In modern space, the important features of an object are those that we can describe as numbers, and a full understanding of that object is a mathematical description of the relationships among those numbers.
- D. So to understand an object (or system) in terms of phase space, we first identify the various properties of the system that we are going to measure. We call each property a parameter, and we assign each parameter to an axis in number space. This space can have as many dimensions as we have parameters.
- E. The *state* of the system at any given moment can, by our previous definitions, be represented by a set of numbers, one number for each parameter. Any given set of numbers can, by the methods of analytic geometry, be understood as a way of indicating a unique point in that space. This number space, which includes a unique point representing every possible state of the system, is called now called the *state space*.
- F. Now, as this system evolves, as it changes over time, those changes will be reflected in changes of the measurable parameters. The

- changed parameters will indicate different points in the state space. We can, thus, represent the change in state of the system as the movement of a point in the state space. The line traced by the moving point in state space is called the *trajectory* of the system. The trajectory of the system is a description of its changing characteristics. The movement of the point in state space need have nothing whatsoever to do with actual movement in physical space.
- G. Now, as we study this trajectory we will, hopefully, begin to discern some pattern in its movement. If the trajectory is, for example, observed to be a straight line, it will be fairly easy to predict what its next value is if we know the previous two values. If the trajectory line is somehow curved, the prediction will become more difficult.
- H. The calculus is a mathematical tool that allows us to predict how the trajectory will unfold in a fairly broad collection of situations. The technicalities of calculus are unimportant for our purpose. What is important is that the calculus allows us to draw, at each point in the state space, a little arrow which points out the direction in which the trajectory will unfold if the trajectory passes through the given point. In other words, we not only have a state space representing every possible set of measurements by which we can characterize the system, we also have a representation of which point the system will go to next if it finds itself at any particular point. The little arrow next to each point is called a *vector*. When each point in the state space has an arrow, the whole space is called the *vectorfield*.
- I. This space, the state space with vectors, is the *phase space* of the system.
- J. As we look at the patterns that are traced out by the arrows in the space, we can begin to notice patterns.
1. Sometimes the arrows from many points will all lead to a single point, and the only arrow leaving that point will point back to itself. In this case that point is called a *point attractor*. A point attractor in phase space represents a steady state of the system, a state into which it will settle if it begins from any of the states that are represented by points with arrows that point towards it.
 2. Sometimes the arrows from many points will all lead to a closed line (e.g., an ellipse), and all of the points on that line will lead to adjacent points on that line. This line is a *periodic attractor*, an oscillating state of the system into which it will settle, given appropriate starting states (appropriate initial conditions).
- K. Chaos theory and complexity theory are mathematical tools for

dealing with special categories of attractors in the phase spaces of systems.

VII.CONCLUSION

- A. This meditation has developed into a rather long and roundabout way of defining phase space.
- B. I believe that the full significance of phase space can only emerge, especially for us humanities students, if phase space is seen in relation to dream space, local space, perspectival space, number space and state space.
- C. I began this meditation with the hope that a deeper understanding of phase space could help us to illuminate both the value and the unwarranted pretensions of chaos theory. As it happened, just reaching a definition of phase space was a much longer journey than I anticipated. Further reflection will have to await a further meditation . . .

NOTES

¹ Einstein Albert and Infeld, Leopold, *The Evolution of Physics*, Simon And Schuster, 1942.

² Einstein and Infeld take this way out. They start their consideration by saying “A most fundamental problem, for thousands of years wholly obscured by its complications, is that of motion.” Einstein and Infeld, *Op. Cit.*, p. 6.

³ This is obvious in the case of the calculus and can probably be established for analytic geometry.

⁴ In the context of this paper, I can barely scratch the surface of the cultural history of space. The notion is explored extensively in Gebser, Jean, *The Ever Present Origin*, Ohio University Press, 1983; in Whorf, Benjamin, *Language, Thought and Reality*, The MIT Press, 1956; and in Romanyshyn, Robert, *Technology as Symptom and Dream*, Routledge, 1989.

⁵ The central ideas of this section come from Romanyshyn, *Op. Cit.* I call the space I am imagining here is ‘mythic space’. Gebser, *Op. Cit.*, uses that term for something analogous to what I am describing, but his notion is much deeper than I could hope to do justice to (or, at this point, to fully fathom).

⁶ I assume that this would be the case in the world we are imagining because stories, as they travel from mouth to mouth over greater distances and longer intervals of time, tend to become more and more archetypal.

⁷ I am using ‘dream’ here in a very broad sense to include all imaginery scenes – all of the spaces in which all of our personal dreams take place, as well as all of the mythic spaces that we find in the collective imagination – e.g., Mt. Olympus, the island of the Cyclops, Camelot, the worlds of *Star Trek*, etc.

⁸ This notion of dream space has, I think, a certain resonance with the Jungian notion of the collective unconscious,

⁹ Phase space is a way of ordering all of the possible measurable states of a system along a set of axes, each of which represents one relevant variable. Dream space is a way of ordering all of the possible imaginary experiences along a set of axes each of which represents one relevant quality. Of course, in dream space, we can hardly imagine that the axes are orthogonal to each other, nor can we clearly imagine how to arrange the various dream spaces along the axes. The point is that each of the spaces is a way of spatially representing the meaningful connections among the various possible states.

¹⁰ I have developed this notion more fully in Weiss, Eric *Some Reflections on the Definition of Synchronicity*, Unpublished, 1998.

¹¹ Gebser, *Op. cit.*

¹² Romanyshyn, *Op. cit.*, p. 40, makes this point in relation to certain features of perspective drawing. I recall Gebser making this point more generally somewhere in *The Everpresent Origin*, but I can’t, at this moment, locate the reference.

¹³ Romanyshyn, *Op. cit.*, Gebser, *Op. cit.* Romanyshyn, writing in the 1980’s, never refers to Gebser, who wrote in the 1940’s. While Romanyshyn’s work is much more limited in scope, in the territory that it does cover it is remarkably consonant with Gebser’s thinking. Presumably, they reached the same conclusions by different routes.

¹⁴ This intuition is explored extensively in Jungian literature. See, especially Von Franz, Marie-Louise, *Number and Time*, Northwestern University Press, 1974 and Von Franz, Marie-Louise, *Psyche and Matter*, Shambhala, 1992.

¹⁵ This definition of phase space comes from Abraham, Ralph and Shaw, Christopher, *Dynamics: The Geometry of Behavior, Part One: Periodic Behavior*, Aerial Press, Inc., Santa Cruz, CA, Undated, pp. 13-21. It would best be read in conjunction with the brilliant diagrams which Abraham and Shaw provide.